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ACOUSTIC PLANE WAVES NORMALLY INCIDENT ON A CLAMPED PANEL IN A RECTANGULAR DUCT

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by

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ABSTRACT

The theory of acoustic plane wave normally incident on a clamped panel in a rectangular duct is developed from basic theoretical concepts. The coupling theory between the elastic vibrations of the panel (plate) and the acoustic wave propagation in infinite space and in the rectangular duct is considered in detail. The partial differential equation which governs the vibration of the panel (plate) is modified by adding to it stiffness (spring) forces and damping forces, and the fundamental resonance frequency f_o and the attenuation factor α are discussed in detail.

The noise reduction expression based on the present theory is found to agree well with the corresponding experimental data of a sample aluminum panel in the mass controlled region ($f > f_o$), the damping controlled region ($f \sim f_o$) and the stiffness controlled region ($f < f_o$). All the frequency positions of the upward and downward resonance spikes in the sample experimental data are identified theoretically as resulting from four cross interacting major resonance phenomena: the cavity resonance, the acoustic resonance, the plate resonance, and the wooden back panel resonance. Detailed tables are given for the values of these resonance frequencies in each case.

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CHAPTER I

INTRODUCTION

The acoustic research project started on April 15, 1976, when the Flight Research Laboratory of the University of Kansas began work under a grant from NASA, Langley Research Center, entitled, "A Research Program to Reduce Interior Noise in General Aviation Airplanes" (NASA Grant No. 1301). Over the past three years a research facility has been established, in the form of a Beranek tube and additional equipment, and a large volume of experimental data has been published. A substantial amount of the experimental data published to date involves the noise reduction curves for acoustic plane waves normally incident on clamped aluminum panels, and other materials, in the Beranek tube.

The aim of the present report is to present a theory which will explain reasonably well the detailed features of the experimental noise reduction curves for normal incidence of acoustic plane waves on a clamped panel in a rectangular duct (the Beranek tube). Such detailed features include the general behavior of the noise reduction curve in all its parts, as well as the frequencies of the numerous resonance spikes, which are superimposed on the curve both upward and downward, for frequencies above the fundamental frequency of resonance. Another aim of the present report is to be used as a theoretical base on which more complicated acoustics experimental results could be studied from a theoretical point of view in the future.

Such a theory sould be based on the interaction between two general fields of study: The theory of acoustic wave propagation in

infinite space and in ducts, and the dynamic theory of plates and the theory of elasticity. Since the vibrations in the panel (plate) and the acoustic waves in the air are coupled strongly along the whole panel, and are affecting each other noticeably, the interaction between these two systems plays a major role in this theory. When one sends an incident wave of sound in the direction of the panel, this wave will be reflected from the panel. At the same time it will set the panel into motion of vibrations, which will generate transmitted acoustic waves on the other side of the panel. The incident, reflected and transmitted acoustic waves will be coupled strongly together with the induced transverse displacement of the panel. Because of the experimental set up in the Beranek tube, the acoustic waves propagate in a duct with a square cross-section. Higher order acoustic modes, as well as the fundamental mode of plane waves, should be taken into account, in order to explain the experimental results of resonance at certain particular frequencies.

The present report develops the theoretical derivations from the basic equations. The theory presented in this report successfully explains the detailed features of the noise reduction curves for one particular aluminum panel, which has been taken as a typical example. It should be pointed out that one of the most challenging aspects of the present theory, which has been met successfully, is to avoid lengthy numerical analysis, in order not to mask the main features of the interaction between the different physical phenomena. All the calculations in the present report have been done on a simple hand calculator.

The Meter, Kilogram, Second system of units is being used throughout this report, except at some places where the experimental data is given otherwise. The factor $e^{-i\omega t}$ is used for harmonic time variation.

In this report the following vector identities will be used, where $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$ are the unit vectors in rectangular coordinates:

$$\nabla p = (\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z})\, p = \bar{a}_x \frac{\partial p}{\partial x} + \bar{a}_y \frac{\partial p}{\partial y} + \bar{a}_z \frac{\partial p}{\partial z}$$

$$\nabla \cdot \bar{u} = (\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z}) \cdot \bar{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

$$\nabla \times \bar{u} = \bar{a}_x(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}) + \bar{a}_y(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}) + \bar{a}_z(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y})$$

$$\nabla^2 p = \nabla \cdot \nabla p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}$$

$$\nabla \times \nabla p = 0 \qquad \nabla \cdot \nabla \times \bar{u} = 0$$

$$\nabla \times (\nabla \times \bar{u}) = \nabla(\nabla \cdot \bar{u}) - \nabla^2 \bar{u}$$

$$\iiint_V \nabla p \, dV = \iint_S p\bar{n}\, dS \qquad \text{closed surface integral}$$

$$\iiint_V \nabla \cdot \bar{u} \, dV = \iint_S \bar{u} \cdot \bar{n}\, dS \qquad \text{closed surface integral}$$

CHAPTER II

ACOUSTIC WAVES - BASIC THEORY

In the present chapter the basic equations of the acoustic waves will be given to be used in this report.

The equation of state for a perfect gas may be given in the form:

$$MP = R_o \rho T \tag{1}$$

where: M = molecular weight of the gas (Kg/mole)

P = the total pressure of the gas (N/m^2) or (J/m^3)

R_o = 8.314×10^3 (J/°Kmole)-gas constant

ρ = the total gas density of the gas (Kg/m^3)

T = temperature in °K where T(°K) = 273.15° + T(°C)

In the particular case of air, which consists of Oxygen ($M_{O_2} \cong 32$ Kg/mole) and Nitrogen ($M_{N_2} \cong 28$ Kg/mole), the molecular weight is:

$$M_{air} = 28.9644 \text{ (Kg/mole)}$$

Substituting this value in Eqn. (1) one obtains the equation of state for air in the form:

$$P = R\rho T \tag{2}$$

where the gas constant for air is given by:

$$R = \frac{R_o}{M} \cong 287.05 \text{ (J/°K Kg)}$$

For example, the U.S. standard atmosphere tables give the following values at sea level at $T = 15°C = 59°F$:

$$P = 760\text{mm Hg} = 1.01325 \times 10^5\ (\text{N/m}^2)\ (1\ \text{atmosphere} = 10^5\ \text{N/m}^2)$$

$$\rho = 1.2250\ (\text{Kg/m}^3)$$

$$T = 273.15° + 15.00° = 288.15°$$

and obey Equation (2) above.

The acoustic wave motion, when the wave amplitude is small, can be described by the following varying quantities:

$$p = p(x,y,z,t) = \text{acoustic or sound pressure } (\text{N/m}^2) \quad \text{where } |p| << P$$

$$\bar{u} = \bar{u}(x,y,z,t) = \text{velocity vector of the air} \quad (\text{m/sec})$$

where P is the equilibrium pressure.

One requires one scalar equation and one vector equation to relate these varying quantities for the acoustic wave motion.

From the equation of continuity one obtains:

$$\kappa \frac{\partial p(x,y,z,t)}{\partial t} = -\nabla \cdot \bar{u}(x,y,z,t) \tag{3}$$

which states that the velocity gradient produces a compression of the air. The only forces involved are those of compressive elasticity, measured by the compressibility κ. Since in the case of the acoustic wave motion there is zero heat conduction in the air, one should use the adiabatic compressibility given as follows:

$$\kappa = \frac{1}{\gamma P} = \text{adiabatic compressibility} \quad (m^2/N) \text{ or } (m^3/J)$$

where P is the equilibrium pressure and for perfect diatomic gas, such as air, one has:

$$\gamma = \frac{7}{5} = 1.4 = \frac{C_p}{C_v} = \frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$$

From the equation of wave motion one obtains:

$$\rho \frac{\partial \bar{u}(x,y,z,t)}{\partial t} = -\nabla p(x,y,z,t) \tag{4}$$

which states that a pressure gradient produces an acceleration of the air, where ρ is the equilibrium density of the air.

Taking the divergence on both sides of Eqn. (4) and substituting Eqn. (3) one obtains:

$$\nabla \cdot \nabla p = -\rho \frac{\partial}{\partial t} (\nabla \cdot \bar{u}) = -\rho \frac{\partial}{\partial t}(-\kappa \frac{\partial p}{\partial t}) = \kappa\rho \frac{\partial^2 p}{\partial t^2}$$

which may be rewritten in the form of the wave equation:

$$\nabla^2 p - \kappa\rho \frac{\partial^2 p}{\partial t^2} = 0 \text{ or } \nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{5}$$

where c is the velocity of the acoustic wave in the air. Substituting $\kappa = \frac{1}{\gamma P}$ in Eqn. (5), using Eqn. (2), one obtains:

$$c = \frac{1}{\sqrt{\kappa\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma RT} = 20.047\sqrt{T(°K)} \qquad (m/sec) \tag{6}$$

where one has:

$$T(°K) = 273.15° + T(C°)$$

For example at T = 15°C (59°F) the acoustic wave velocity is:

$$c = 340.3 \text{ m/sec} \qquad \text{for } T = 15°C(59°F)$$

Taking the gradient on both sides of Eqn. (3) and substituting Eqn. (4) one obtains:

$$\nabla\nabla \cdot \bar{u} = -\kappa\frac{\partial}{\partial t}(\nabla p) = -\kappa\frac{\partial}{\partial t}\left[-\rho\frac{\partial \bar{u}}{\partial t}\right] = \kappa\rho\frac{\partial^2 \bar{u}}{\partial t^2}$$

Since from Eqn. (4) one has for the time varying acoustic wave $\nabla \times \bar{u} = 0$, one obtains:

$$\nabla\nabla \cdot \bar{u} - \nabla^2\bar{u} = \nabla \times \nabla \times \bar{u} = 0 \qquad \text{since } \nabla \times \bar{u} = 0$$

Substituting it in the above, one has the wave equation for the veolcity vector field:

$$\nabla^2\bar{u} - \kappa\rho\frac{\partial^2 \bar{u}}{\partial t^2} = 0 \quad \text{or } \nabla^2\bar{u} - \frac{1}{c^2}\frac{\partial^2 \bar{u}}{\partial t^2} = 0 \qquad (7)$$

In order to find the transmitted power in the acoustic wave motion it is necessary to develop a power theorem for the acoustic waves, using the vector identity:

$$\nabla \cdot (p\bar{u}) = p\nabla \cdot \bar{u} + \bar{u} \cdot \nabla p \tag{8}$$

By multiplying both sides of Eqn. (3) by p and dot-product both sides of Eqn. (4) by $\bar{u}$ and substituting in Eqn. (8) one obtains:

$$\nabla \cdot (p\bar{u}) = -\kappa p\frac{\partial p}{\partial t} - \rho\bar{u} \cdot \frac{\partial \bar{u}}{\partial t} \tag{9}$$

It may be shown that:

$$\kappa p\frac{\partial p}{\partial t} = \kappa\frac{1}{2}\frac{\partial}{\partial t}(pp) = \frac{\partial}{\partial t}(\frac{1}{2}\kappa p^2) \tag{10}$$

$$\rho\bar{u} \cdot \frac{\partial \bar{u}}{\partial t} = \rho\frac{1}{2}\frac{\partial}{\partial t}(\bar{u} \cdot \bar{u}) = \frac{\partial}{\partial t}(\frac{1}{2}\rho|\bar{u}|^2) = \frac{\partial}{\partial t}(\frac{1}{2}\rho u^2) \tag{11}$$

Substituting Eqn. (10 and Eqn. (11) in Eqn. (9), one obtains:

$$\nabla \cdot (p\bar{u}) = -\frac{\partial}{\partial t}(\frac{1}{2}\kappa p^2) - \frac{\partial}{\partial t}(\frac{1}{2}\rho u^2) \tag{12}$$

Integrating both sides in an arbitrary volume V and using the Gauss divergence theorem, one obtains

$$\iiint_V \nabla \cdot (p\bar{u})dV = \iint_S (p\bar{u}) \cdot \bar{n}dS = -\frac{\partial}{\partial t}\iiint_V [\frac{1}{2}\kappa p^2 + \frac{1}{2}\rho u^2]dV \tag{13}$$

Physically Eqn. (13) states that the total radiated (transmitted) acoustic wave power through the surface S is equal to the total decrease of acoustic wave power inside the volume V, where we have:

$w_P = \frac{1}{2}\kappa p^2$ = acoustic wave potential energy density (J/m^3)

$w_K = \frac{1}{2}\rho u^2$ = acoustic wave kinetic energy density (J/m^3)

$\bar{I} = p\bar{u}$ = instantaneous radiated (transmitted) acoustic wave power density (Watt/m^2) or (J/sec m^2)

Thus the radiated (transmitted) acoustic wave power is in the direction of the acoustic wave velocity vector $\bar{u}$. The vector $\bar{I}$ is sometimes called the "sound intensity." Taking the total acoustic wave energy to be $w = w_P + w_K$, one can rewrite Eqn. (12) in the form:

$$\nabla \cdot \bar{I} + \frac{\partial}{\partial t}(w_P + w_K) = \nabla \cdot \bar{I} + \frac{\partial}{\partial t}w = 0 \tag{14}$$

where Eqn. (14) gives us the nondissipative energy equation of continuity for the acoustic wave with no sources in the vicinity.

By using the Gauss divergence theorem, Eqn. (3) may be rewritten in an integrated form as follows:

$$-\iiint_V \kappa \frac{\partial p}{\partial t} dV = -\frac{d}{dt}\iiint_V \kappa p dV = \iiint_V \nabla \cdot \bar{u} dV = \iint_S \bar{u} \cdot \bar{n} dS \tag{15}$$

where the right hand integral gives the total velocity flux.
By taking a small cylinder of area (ΔS) and height(Δz) across the boundary between two different fluid media, applying the integrals in Eqn. (15) and taking $\Delta z \to 0$, one obtains the following boundary condition:

$$u_{n_1} = u_{n_2} \tag{16}$$

The normal component of the velocity vector of the acoustic wave is continuous across the boundary.

By using the corresponding vector theorem, Eqn. (4) may be rewritten in an integral form as follows:

$$-\iiint\limits_V \rho \frac{\partial \bar{u}}{\partial t}\, dV = -\frac{d}{dt}\iiint\limits_V \rho \bar{u}\, dV = \iiint\limits_V \nabla p\, dV = \iint\limits_S p\bar{n} dS \tag{17}$$

where the integral $\iiint\limits_V \rho\bar{u}dV$ gives the total momentum in the volume. Similarly to the above, one obtains the following boundary condition:

$$p_1 = p_2 \tag{18}$$

The pressure of the acoustic wave is continuous across the boundary.

CHAPTER III

ACOUSTIC UNIFORM PLANE WAVE

In the present chapter the acoustic waves basic equations will be applied to the harmonically varying acoustic uniform plane wave in infinite fluid (air) medium where the effects of viscosity, heat conduction, relaxation, vibration and other dissipative (attenuation) processes have been neglected.

Assuming harmonic time variation $e^{-i\omega t}$, where ω is the circular frequency, one may write the basic quantities of the acoustic wave by using phasor convention as follows:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\,\underset{\sim}{p}(x,y,z)e^{-i\omega t} \tag{19a}$$

$$\bar{u}(x,y,z,t) = \mathrm{Re}\sqrt{2}\,\bar{\underset{\sim}{u}}(x,y,z)e^{-i\omega t} \tag{19b}$$

where Re represents the real part and $\underset{\sim}{p}(x,y,z)$ and $\bar{\underset{\sim}{u}}(x,y,z)$ are the complex effective value phasors, which represent both amplitude and phase in the complex plane. For example one has:

$$\underset{\sim}{p} = |\underset{\sim}{p}|e^{i\phi} \qquad p = \mathrm{Re}\sqrt{2}|\underset{\sim}{p}|e^{i\phi}e^{-i\omega t} = \sqrt{2}\,|\underset{\sim}{p}|\cos(\omega t - \phi)$$

where $|\underset{\sim}{p}|$ is the effective value, with similar relations for each component of the complex velocity vector $\bar{\underset{\sim}{u}}(x,y,z)$.

In order to simplify the notation, $\underset{\sim}{p}(x,y,z)$ and $\bar{\underset{\sim}{u}}(x,y,z)$, will be denoted from now on as $p = p(x,y,z)$ and $\bar{u} = \bar{u}(x,y,z)$, where the harmonic time variation convention of Eqn. (19) is understood.

Assuming harmonic time variation $e^{-i\omega t}$ and replacing $\frac{\partial}{\partial t}$ by $(-i\omega)$, the two basic equations Eqns. (3) and (4) for the acoustic waves for harmonic time variation will become:

$$i\omega\kappa p = \nabla \cdot \bar{u} \tag{20a}$$

$$i\omega\rho\bar{u} = \nabla p \tag{20b}$$

where complex quantities $p = \underset{\sim}{p}$ and $\bar{u} = \underset{\sim}{\bar{u}}$ in Eqn. (20) are understood. From (20b) one also obtains:

$$\nabla \times \bar{u} = 0 \tag{20c}$$

Thus, $\bar{u}(x,y,z)$ may be found from a given $p(x,y,z)$ in Eqn. (20b) and $p(x,y,z)$ may be found from a given $\bar{u}(x,y,z)$ in Eqn. (20a). By taking the divergence of both sides of Eqn. (20b) and substituting Eqn. (20a), one obtains:

$$\nabla \cdot \nabla p = i\omega\rho\nabla \cdot \bar{u} = i\omega\rho(i\omega\kappa p) = -(\omega^2\kappa\rho)p$$

which may be written in the form of the wave equation for harmonic time variation:

$$\nabla^2 p + (\omega^2\kappa\rho)p = 0 \quad \text{or } \nabla^2 p + k^2 p = 0 \tag{21a}$$

By taking the gradient of both sides of Eqn. (20a) and substituting Eqn. (20b) one obtains

$$\nabla(\nabla \cdot \bar{u}) = i\omega\kappa\nabla p = i\omega\kappa(i\omega\rho\bar{u}) = -(\omega^2\kappa\rho)\bar{u}$$

By using Eqn. (20c) it may be written in the form of the wave equation for harmonic time variation:

$$\nabla^2\bar{u} + (\omega^2\kappa\rho)\bar{u} = 0 \qquad \text{or} \qquad \nabla^2\bar{u} + k^2\bar{u} = 0 \tag{21b}$$

The wave number k is defined by:

$$k = \omega\sqrt{\kappa\rho} = \frac{\omega}{c} = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda} \quad (1/m) \tag{22}$$

where: ω = circular frequency (1/sec)

$\frac{\omega}{2\pi}$ = f = the frequency of the acoustic wave (1/sec)

λ = the wavelength of the acoustic wave (m)

$f\lambda$ = c = the velocity of the acoustic wave (m/sec)

Assuming acoustic uniform plane wave propagating in the z-direction one has p = p(z) and Eqn. (21a) becomes:

$$\frac{d^2p}{dz^2} + k^2p = 0 \qquad p = p_o e^{ikz} \qquad \text{or} \qquad p = p_o e^{-ikz}$$

where p_o is a constant.

Substituting the first solution in Eqn. (19a) one obtains:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\, p_o e^{i(kz-\omega t)} = \sqrt{2}\, p_o \cos(kz-\omega t) \qquad (23a)$$

Substituting Eqn. (23a) in Eqn. (20b), one finds $u_x = 0$, $u_y = 0$, $u_z \neq 0$. By solving Eqn. (21b), one obtains similarly:

$$u_z(x,y,z,t) = \mathrm{Re}\sqrt{2} u_o e^{i(kz-\omega t)} = \sqrt{2}\, u_o \cos(kz-\omega t) \qquad (23b)$$

Substituting $p = p_o e^{ikz}$ and $u_z = u_o e^{ikz}$ in Eqn. (20a) and Eqn. (20b) one obtains by using Eqn. (22):

$$\frac{p}{u_z} = \frac{p_o}{u_o} = \frac{k}{\omega\kappa} = \frac{\omega\rho}{k} = \sqrt{\frac{\rho}{\kappa}} = \frac{1}{\kappa c} = \rho c = Z_o \qquad (23c)$$

where Z_o is the characteristic acoustic wave impedance in infinite medium in $(\mathrm{Nsec/m^3})$ or $(\mathrm{kg/m^2 sec})$.

The characteristic acoustic wave impedance for air at sea level at 15°C(59°F) will be:

$$Z_o = \rho c = (1.2250\ \mathrm{Kg/m^3}) \cdot (340.3\ \mathrm{m/sec}) = 417\ \frac{\mathrm{Nsec}}{\mathrm{m^3}}$$

Equations (23) represent the acoustic uniform plane wave propagating in the positive z-direction. The pressure is in phase with the fluid velocity. It is a longitudinal wave since the velocity vector $\bar{u}$ has only one component in the direction of propagation.

Similarly, the acoustic uniform plane wave propagating in the negative z-direction will have to form:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\, p_o e^{-i(kz+\omega t)} = \sqrt{2} p_o \cos(kz+\omega t) \tag{24a}$$

$$u_z(x,y,z,t) = -\mathrm{Re}\sqrt{2} u_o e^{-i(kz+\omega t)} = -\sqrt{2} u_o \cos(kz+\omega t) \tag{24b}$$

where $u_o = \dfrac{p_o}{Z_o} = \dfrac{p_o}{\rho c}$

The velocity vector u_z is again directed in the direction of propagation of the acoustic plane wave.

The average acoustic plane wave potential energy density is given by:

$$w_P = \frac{\kappa}{2T} \int_{t=0}^{T} [\sqrt{2} p_o \cos(kz-\omega t)]^2 dt = \frac{\kappa}{2} p_o^2 = \frac{\kappa}{2} |p|^2 \quad (\mathrm{J/m^3}) \tag{25a}$$

where the integral has been evaluated by taking $\omega = 2\pi f = \dfrac{2\pi}{T}$

where T = the period (sec)

Similarly, the average acoustic plane wave kinetic energy density is given by:

$$w_K = \frac{\rho}{2T} \int_{t=0}^{T} [\sqrt{2} u_o \cos(kz-\omega t)]^2 dt = \frac{\rho}{2} u_o^2 = \frac{\rho}{2} |u_z|^2 \quad (\mathrm{J/m^3}) \tag{25b}$$

From Eqn. (23c) we have $\kappa p_o^2 = \rho u_o^2$ and therefore in the acoustic uniform plane wave one has $w_P = w_K$. The average total acoustic wave energy density is given by:

$$w = w_P + w_K = \kappa |p|^2 = \rho |u_z|^2 = \frac{|p|^2}{\rho c^2} \quad (\mathrm{J/m^3}) \tag{25c}$$

The average radiated (transmitted) acoustic plane wave power (the "sound intensity") is given by:

$$I_z = 2p_o u_o \frac{1}{T}\int_{t=0}^{T} \cos^2(kz-\omega t)dt = p_o u_o = \frac{|p|^2}{\rho c} = \rho c|u_z|^2 =$$

$$= cw = \frac{1}{2}(p^* u_z + p u_z^*) \qquad (\text{Watt/m}^2) \qquad (26)$$

where p^* is the complex conjugate of p.

Let an acoustic plane wave propagating in the positive z-direction be incident on a rigid wall at z = L. As a result an acoustic wave will be reflected from the wall propagating in the negative z-direction. The incident plane wave is given by using the phasor convention as follows:

$$p_i = p_I e^{ikz} \qquad u_{zi} = \frac{1}{\rho c} p_I e^{ikz} \qquad (27a)$$

where p_I is a complex constant.

The reflected acoustic plane wave is given by:

$$p_r = p_R e^{-ikz} \qquad u_{zr} = -\frac{1}{\rho c} p_R e^{-ikz} \qquad (27b)$$

where p_R is a complex constant.

At the rigid wall z = L one has $u_n = 0$:

$$u_n = [u_{zi} + u_{zr}]_{z=L} = \frac{1}{\rho c}[p_I e^{ikL} - p_R e^{-ikL}] = 0 \qquad (28a)$$

which gives:

$$p_R = p_I e^{i2kL} \tag{28b}$$

The total pressure and velocity fields of both waves will be therefore:

$$p = p_i + p_r = p_I e^{ikz} + p_I e^{i2kL} e^{-ikz} \tag{29a}$$

$$u_z = u_{zi} + u_{zr} = \frac{1}{\rho c} p_I e^{ikz} - \frac{1}{\rho c} p_I e^{i2kL} e^{-ikz} \tag{29b}$$

where at $z = L$ one has $u_z = u_n = 0$.

Multiplying by $\sqrt{2}\, e^{-i\omega t}$ and taking the real part of it in accordance with Eqns. (19), one finds the actual physical fields:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\, p_I\, [e^{ikz} + e^{i2kL} e^{-ikz}] e^{-i\omega t} =$$

$$= \mathrm{Re}\sqrt{2}\, p_I [e^{i(kz-kL)} + e^{-i(kz-kL)}]\, e^{-i(\omega t-kL)} =$$

$$= 2\sqrt{2}\, p_I \cos(kz-kL)\cos(\omega t-kL) \tag{30a}$$

$$u_z(x,y,z,t) = \mathrm{Re}\sqrt{2}\, \frac{1}{\rho c}\, p_I [e^{ikz} - e^{i2kL}\, e^{-ikz}]\, e^{-i\omega t} =$$

$$= \mathrm{Re}\sqrt{2}\, \frac{1}{\rho c}\, p_I\, [e^{i(kz-kL)} - e^{-i(kz-kL)}]\, e^{-i(\omega t-kL)} =$$

$$= \mathrm{Re}\sqrt{2}\, \frac{1}{\rho c}\, p_I 2i \sin(kz-kL)\, e^{-i(\omega t-kL)} =$$

$$= 2\sqrt{2}\, \frac{p_I}{\rho c} \sin(kz-kL)\sin(\omega t-kL) \tag{30b}$$

Both Eqn. (30a) and Eqn. (30b) represent standing waves. One may find the maximum value of the pressure from Eqn. (30a) to be at any point $z = z_o$.

$$p_{max}(z=z_o) = 2\sqrt{2}\, p_I \left|\cos k(z_o-L)\right| \tag{31a}$$

Eqn. (31a) may be found alternatively directly from Eqn. (29a) as follows:

$$\begin{aligned} p_{max}(z=z_o) &= \sqrt{2}\, p_I \left| e^{ikz_o} + e^{i2kL} e^{-ikz_o} \right| = \\ &= \sqrt{2}\, p_I \left|e^{ikL}\right| \left|e^{ik(z_o-L)} + e^{-ik(z_o-L)}\right| = 2\sqrt{2}\, p_I \left|\cos k(z_o-L)\right| \end{aligned} \tag{31b}$$

Let us consider the case when the wall at $z = L$ is not rigid. We will define the non-dimensional specific acoustic impedance ξ of the wall at $z = L$ as follows:

$$\left.\frac{p}{\rho c\, u_z}\right|_{z=L} = \frac{Z}{\rho c} = \zeta = \theta - i\chi \qquad \text{(dimensionless)} \tag{32a}$$

Where θ is the specific acoustic resistance and χ is the specific acoustic reactance. Substituting Eqn. (27) in Eqn. (32a), one obtains:

$$\frac{p_I e^{ikL} + p_R e^{-ikL}}{p_I e^{ikL} - p_R e^{-ikL}} = \zeta \tag{32b}$$

From Eqn. (32b) one obtains the complex reflection coefficient C_r:

$$C_r = \frac{p_R}{p_I} = \frac{\zeta-1}{\zeta+1}\, e^{i2kL} \tag{32c}$$

For the particular case of a rigid wall $\zeta = -i\infty$ one obtains Eqn. (28b). For the particular case of a purely reactive surface $\zeta = -i\chi$ one has $|C_r| = 1$ and:

$$C_r = \frac{p_R}{p_I} = \frac{-i\chi-1}{-i\chi+1} e^{i2kL} = e^{-i2\phi} e^{i2kL} = e^{i2(kL-\phi)} \tag{33}$$

The reflected wave differs only in the phase angle from the incident wave; the reflected acoustic power is equal to the incident acoustic power; therefore no energy is absorbed by the surface and one has standing waves as before. For the particular case of the rigid wall $\chi = \infty$ and $\phi = 0$. The total pressure and velocity fields of both waves will be therefore:

$$p = p_i + p_r = p_I e^{ikz} + p_I e^{i2(kL-\phi)} e^{-ikz} \tag{34a}$$

$$u_z = u_{zi} + u_{zr} = \frac{1}{\rho c} p_I e^{ikz} - \frac{1}{\rho c} p_I e^{i2(kL-\phi)} e^{-ikz} \tag{34b}$$

One may find the maximum value of the pressure from Eqn. (34a) to be at any point $z = z_o$.

$$p_{max} \quad (z=z_o) = \sqrt{2}\, p_I \left| e^{ikz_o} + e^{i2(kL-\phi)} e^{-ikz_o} \right| =$$

$$= \sqrt{2}\, p_I \left| e^{i(kL-\phi)} \right| \left| e^{i[kz_o-(kL-\phi)]} + e^{-i[kz_o-(kL-\phi)]} \right| =$$

$$= 2\sqrt{2}\, p_I \left| \cos[k(z_o - L) + \phi] \right| =$$

$$= 2\sqrt{2}\, p_I \left| \cos[k(L - z_o) - \phi] \right| \tag{35}$$

If the acoustic impedance of the wall is a function of frequency, one has $\zeta = \zeta(\omega)$ and $\chi = \chi(\omega)$, and one will have in Eqn. (35) $\phi = \phi(\omega)$.

CHAPTER IV

ACOUSTIC WAVES IN RECTANGULAR DUCTS

In the present chapter the guided acoustic waves in a rectangular duct (tube) with rigid walls will be developed. By a guided acoustic wave it is meant that the direction of the acoustic energy flow must be primarily along the direction of the guiding system of the duct. The analysis will be limited here to the guiding system of the duct which is straight and has a uniform rectangular cross-section and rigid walls.

The basic equations for the acoustic wave motion are given in Equations (3), (4) and (5) as follows:

$$\kappa\frac{\partial p}{\partial t} = -\nabla \cdot \bar{u} \tag{36a}$$

$$\rho\frac{\partial \bar{u}}{\partial t} = -\nabla p \tag{36b}$$

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad \text{where} \qquad c = \frac{1}{\sqrt{\kappa\rho}} \tag{36c}$$

The acoustic wave propagating along the uniform guiding system of the duct in the z-direction can be described in terms of a propagation factor $e^{i(qz-\omega t)}$, where we have:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\ \underset{\sim}{p}(x,y)e^{i(qz-\omega t)} \qquad (\mathrm{N/m^2}) \tag{37a}$$

$$\bar{u}(x,y,z,t) = \mathrm{Re}\sqrt{2}\ \underset{\sim}{\bar{u}}(x,y)e^{i(qz-\omega t)} \qquad (\mathrm{m/sec}) \tag{37b}$$

where $\underset{\sim}{p}(x,y)$ is a complex scalar phasor and $\underset{\sim}{\bar{u}}(x,y)$ is a complex vector described by three scalar phasors $\underset{\sim}{u}_x(x,y)$, $\underset{\sim}{u}_y(x,y)$ and $\underset{\sim}{u}_z(x,y)$. Substituting Eqn. (37) in Eqn. (36) one obtains:

$$i\omega\kappa p = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + iqu_z \tag{38a}$$

$$i\omega\rho u_x = \frac{\partial p}{\partial x}; \quad i\omega\rho u_y = \frac{\partial p}{\partial y}; \quad i\omega\rho u_z = iqp \tag{38b}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + (k^2 - q^2)p = 0 \tag{38c}$$

where $p = \underset{\sim}{p}(x,y)$, $u_x = \underset{\sim}{u}_x(x,y)$, $u_y = \underset{\sim}{u}_y(x,y)$, $u_z = \underset{\sim}{u}_z(x,y)$ and

$$k = \frac{\omega}{c} = \frac{\omega}{\sqrt{\kappa\rho}} = \frac{2\pi f}{c} = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda} \qquad (1/m) \tag{38d}$$

k being the infinite medium wave number. By solving the wave Eqn. (38c) one obtains p, and substituting in Eqn. (38b) one obtains $\bar{u}$.

Substituting in Eqn. (38c):

$$k_c^2 = k^2 - q^2 \tag{39a}$$

one obtains:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + k_c^2 p = 0 \tag{39b}$$

Eqn. (39a) may be rewritten in the form:

$$q = \sqrt{k^2 - k_c^2} \qquad (1/m) \tag{40a}$$

where q is the duct wave number, k is the infinite medium wave number and k_c is the cut off wave number. When $k = k_c$ one has $q = 0$ and there is no propagation. It should be pointed out that q could be either positive for guided waves propagating in the positive z-direction, or negative for guided waves propagating in the negative z-direction. The cut off wave number k_c is defined by:

$$k_c = \frac{\omega_c}{c} = \frac{2\pi f_c}{c} = \frac{2\pi f_c}{f_c \lambda_c} = \frac{2\pi}{\lambda_c} \qquad (1/m) \qquad (40b)$$

where f_c is the cut off frequency and λ_c is the wavelength of an acoustic uniform plane wave at the cut off frequency f_c in infinite medium . One obtains from Eqn. (40a):

$$q = k\sqrt{1-(\frac{k_c}{k})^2} = k\sqrt{1 - (\frac{f_c}{f})^2} = k\sqrt{1 - (\frac{\lambda}{\lambda_c})^2} \quad (1/m) \quad (40c)$$

since $c = f\lambda = f_c\lambda_c$. From Eqn. (40c) it is found that when $f > f_c$ the duct wave number q is real and the acoustic wave propagates without attenuation along the duct. When $f < f_c$, the duct wave number q is purely imaginary and the acoustic wave attenuates exponentially and does not propagate in the duct. When $f = f_c$, one has $q = 0$ and there is neither phase shift nor attenuation along the duct and this condition is called the cut off of the mode. The duct effects the acoustic wave of a particular mode of propagation as a high pass filter. The acoustic wave of a particular mode propagates in the duct only if its frequency f is larger than a certain cut off frequency f_c, such that $f > f_c$.

For the propagating range $f > f_c$ the phase velocity V_p of the acoustic wave mode propagating in the duct is given from Eqn. (38d) and Eqn. (40c) as follows:

$$V_p = \frac{\omega}{q} = \frac{c}{\sqrt{1 - (\frac{f_c}{f})^2}} = \frac{c}{\sqrt{1 - (\frac{\lambda}{\lambda_c})^2}} \quad \text{(m/sec)} \quad f > f_c \qquad (41a)$$

where one has for the phase velocity V_p of the wave in the duct $V_p \geq c$. For the propagating range $f > f_c$ the group velocity V_g of the acoustic wave mode propagating in the duct is given fron Eqn. (41a) as follows:

$$V_g = \frac{d\omega}{dq} = \frac{V_p}{1 - \frac{f}{V_p}\frac{dV_p}{df}} = c\sqrt{1 - (\frac{f_c}{f})^2} = c\sqrt{1 - (\frac{\lambda}{\lambda_c})^2} \quad \text{(m/sec)} \quad f > f_c \quad (41b)$$

where $V_g \leq c$. From Eqn. (41a) and Eqn. (41b) one has:

$$c = \sqrt{V_p V_g} \qquad (41c)$$

The wavelength measured along the acoustic guided wave in the duct in the z-direction is the distance represented by a phase shift of 2π, and is denoted by λ_g:

$$\lambda_g = \frac{2\pi}{q} = \frac{\lambda}{\sqrt{1-(\frac{f_c}{f})^2}} = \frac{\lambda}{\sqrt{1 - (\frac{\lambda}{\lambda_c})^2}} \quad \text{(m)} \qquad (41d)$$

The wave number q for the propagating mode in the duct for $f > f_c$ can be expressed as follows:

$$q = \frac{\omega}{V_p} = \frac{2\pi f}{V_p} = \frac{2\pi f}{f\lambda_g} = \frac{2\pi}{\lambda_g} \quad \text{(1/m)} \qquad (42)$$

The acoustic wave impedance in the duct Z_d is defined similarly to the case of the impedance of the acoustic plane wave in the infinite medium Z_o from Eqns. (38b) and (40c) as follows:

$$Z_d = \frac{p}{u_z} = \frac{\omega\rho}{q} = \frac{\rho c}{\sqrt{1 - (\frac{f_c}{f})^2}} = \frac{Z_o}{\sqrt{1 - (\frac{f_c}{f})^2}} \quad (\mathrm{Nsec/m^3}) \text{ or } (\mathrm{Kg/m^2 sec}) \qquad (43)$$

where $Z_d \geq Z_o$. For the propagating acoustic mode $f > f_c$, Z_d is a real number, and there is a transfer of average acoustic power in the duct. For the attenuating acoustic mode $f < f_c$, Z_d is an imaginary number, and there is no transfer of average acoustic power in the duct.

Taking $f_c/f = \cos\theta$ one may obtain the following relations for the acoustic wave mode of propagation from Eqns. (40),(41),(42) and (43):

$$\frac{f_c}{f} = \frac{\lambda}{\lambda_c} = \cos\theta \qquad (44a)$$

$$q = k \sin\theta \qquad (44b)$$

$$V_p = \frac{c}{\sin\theta} \qquad V_g = c \sin\theta \qquad (44c)$$

$$\lambda_g = \frac{\lambda}{\sin\theta} \qquad Z_d = \frac{Z_o}{\sin\theta} \qquad (44d)$$

where one would consider the mode of propagation of the acoustic wave in the duct as an oblique multiple reflection of the acoustic plane wave between two parallel plane boundaries, where the angle of incidence is θ with the normal to the boundary.

The acoustic waves in the duct with the uniform rectangular cross section propagate in the z-direction. The boundaries (walls) of the inside rectangular cross section of the duct of dimensions "a" and "b" are defined by x = 0, x = a, y = 0 and y = b. Assuming that the boundaries (walls) of the duct are rigid, one should have the normal velocity of the acoustic wave fluid to be $u_n = 0$ to each one of the walls. Mathematically, the boundary conditions of the walls of the duct are expressed as follows:

$$u_x = 0 \qquad x = 0; \qquad u_x = 0 \qquad x = a \tag{45a}$$

$$u_y = 0 \qquad y = 0; \qquad u_y = 0 \qquad y = b \tag{45b}$$

Using Eqn. (38b) in Eqn. (45), one may express the boundary conditions as follows:

$$\frac{\partial p}{\partial x} = 0 \qquad x = 0 \qquad \text{and} \qquad x = a \tag{46a}$$

$$\frac{\partial p}{\partial y} = 0 \qquad y = 0 \qquad \text{and} \qquad y = b \tag{46b}$$

The partial differential Eqn. (36b) should be solved now, subject to the boundary conditions given in Eqn. (46). Using the method of separation of variables, one may assume a solution of Eqn. (39b) in the form:

$$p(x,y) = X(x)\ Y(y) \tag{47a}$$

Substituting it in Eqn. (39b) and dividing by X(x) Y(y), one obtains:

$$\frac{1}{X(x)} \frac{d^2X(x)}{dx^2} + \frac{1}{Y(y)} \frac{d^2Y(y)}{dy^2} + k_c^{\ 2} = 0 \tag{47b}$$

This equation is to hold for all values of x and y; since x and y may change independently of each other, then one should have each term to be a constant:

$$\frac{1}{X} \frac{d^2X(x)}{dx^2} = -k_x^{\ 2}; \qquad \frac{1}{Y(y)} \frac{d^2Y(y)}{dy^2} = -k_y^{\ 2} \tag{47c}$$

$$k_c^{\ 2} = k_x^{\ 2} + k_y^{\ 2} \tag{47d}$$

Solving Eqn. (47c), one obtains the general solutions:

$$X(x) = A \cos k_x x + B \sin k_x x$$

$$Y(y) = C \cos k_y y + D \cos k_y y$$

Applying the boundary conditions given in Eqn. (46), one obtains the solutions in the form:

$$X(x) = A \cos k_x x \qquad k_x = \frac{m\pi}{a} \tag{48a}$$

$$Y(y) = C \cos k_y y \qquad k_y = \frac{n\pi}{b} \tag{48b}$$

where m and n are independent positive integers or zero.

From Eqns. (37a), (47a) and (48), one obtains the expression for the pressure field of the acoustic wave in the rectangular rigid duct with rectangular cross section:

$$p(x,y,z,t) = \mathrm{Re}\sqrt{2}\, p_o \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{i(qz-\omega t)} =$$

$$= \sqrt{2}\, p_o \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \cos(qz-\omega t) \qquad (49a)$$

By using Eqn. (49a) in Eqn. (38b), one obtains the expressions for the velocity vector field components of the acoustic wave in the rectangular rigid duct with the rectangular cross section:

$$u_x(x,y,z,t) = \mathrm{Re}\sqrt{2}\, i\, \frac{m\pi}{\omega\rho a} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{i(qz-\omega t)} =$$

$$= -\sqrt{2}\, \frac{m\pi}{\omega\rho a} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \sin(qz-\omega t) \qquad (49b)$$

$$u_y(x,y,z,t) = \mathrm{Re}\sqrt{2}\, i\, \frac{n\pi}{\omega\rho b} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, e^{i(qz-\omega t)} =$$

$$= -\sqrt{2}\, \frac{n\pi}{\omega\rho b} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sin(qz-\omega t) \qquad (49c)$$

$$u_z(x,y,z,t) = \mathrm{Re}\sqrt{2}\, \frac{q}{\omega\rho}\, p_o \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{i(qz-\omega t)} =$$

$$= \mathrm{Re}\sqrt{2}\, \frac{p_o}{Z_d} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}\, e^{i(qz-\omega t)} =$$

$$= \sqrt{2}\, \frac{p_o}{Z_d} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \cos(qz-\omega t) \qquad (49d)$$

The Eqns. (49) give the pressure field and the velocity field of the acoustic wave of mode (m,n) in the uniform rectangular duct of rigid walls. The fields obey the boundary conditions in Eqn. (45) and Eqn. (46). The velocity field component u_z given in Eqn. (49d) is in phase with

the pressure field given in Eqn. (49a) and represents a transmission of average acoustic power in the z-direction for a propagating wave $f > f_c$ in accordance with Eqn. (26). The velocity field components u_x in Eqn. (49b) and u_y in Eqn. (49c) are 90° out of phase with the pressure field given in Eqn. (49a) and there is no transmission of average acoustic power in the x-direction or in the y-direction for a propagating wave $f > f_c$ since by Eqn. (26):

$$I_x \propto \frac{1}{T}\int_{t=0}^{T} \cos(qz-\omega t)\,\sin(qz-\omega t)\,dt = \frac{1}{2T}\int_{t=0}^{T} \sin 2(qz-\omega t)\,dt = \left.\frac{\cos 2(qz-\omega t)}{4T\omega}\right|_{t=0}^{T} = 0$$

since $f = \frac{1}{T}$ and $\omega T = 2\pi f T = 2\pi$.

For the case of $f < f_c$ the mode (m,n) of the acoustic wave attenuates exponentially since q will be a purely imaginary number as seen from Eqn. (40c). In this case u_z in Eqn. (49d) will be also 90° out of phase with the pressure p in Eqn. (49a) and there will be no transmission of average acoustic wave power in the z-direction. Thus in case $f < f_c$, the acoustic wave energy in this mode (m,n) will not propagate, but will oscillate back and forth near the source of the acoustic wave power.

From Eqns. (47d) and (48) one has for the acoustic wave mode (m,n) in the duct:

$$k_c = (k_c)_{m,n} = \sqrt{k_x^{\,2} + k_y^{\,2}} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad (1/m) \quad (50a)$$

From Eqn. (40a) and Eqn. (50a) one has:

$$q = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} = \sqrt{(\frac{\omega}{c})^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} \quad (1/m) \quad (50b)$$

From Eqn. (40b) and Eqn. (50a), one has the cut off frequency f_c and the cut off wavelength λ_c for the acoustic wave mode (m,n) in the duct:

$$f_c = (f_c)_{mn} = \frac{c}{2\pi} k_c = \frac{c}{2}\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} \qquad (1/sec) \qquad (50c)$$

$$\lambda_c = (\lambda_c)_{mn} = \frac{2\pi}{k_c} = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}} = \frac{2ab}{\sqrt{(mb)^2 + (na)^2}} \quad (m) \qquad (50d)$$

where $c = f_c \lambda_c$

There are a doubly infinite possible number of discrete acoustic wave modes corresponding to all the combinations of the integers m and n. From Eqn. (49) one sees that m represents the number of half-sine variations in the x-direction and n represents the number of the half-sine variations of the acoustic wave field components in the y-direction. Thus, the acoustic wave components of the mode (m,n) in the rectangular duct with rigid walls will be denoted by p_{mn} and $\bar{u}_{mn}$ and is given in Eqns. (49).

Of particular interest is the mode (m = 0, n = 0) which is referred to as the fundamental mode or the dominant mode of the acoustic wave in the rectangular duct with rigid walls. Substituting in Eqns. (50), m = 0 and n = 0, one obtains for the fundamental dominant mode:

$$(k_c)_{00} = 0; \qquad (f_c)_{00} = 0; \qquad (\lambda_c)_{00} = \infty \qquad (51a)$$

From Eqns. (40), (41), (43) and (44), one obtains for m = 0 and n = 0:

$$q_{00} = k = \frac{\omega}{c}; \quad (V_p)_{00} = (V_g)_{00} = c; \quad (\lambda_g)_{00} = \lambda; \ (Z_d)_{00} = \rho c = Z_o;$$

$$\theta = \frac{\pi}{2} \tag{51b}$$

Taking m = 0 n = 0 and $q_{00} = k$ in Eqn. (49), one obtains:

$$p_{00}(x,y,z,t) = \mathrm{Re}\sqrt{2}\, p_o e^{i(kz-\omega t)} = \sqrt{2}\, p_o \cos(kz-\omega t) \tag{51c}$$

$$u_{z00}(x,y,z,t) = \mathrm{Re}\sqrt{2}\, \frac{k}{\omega\rho}\, p_o e^{i(kz-\omega t)} = \sqrt{2}\, \frac{1}{\rho c}\, p_o \cos(kz-\omega t) \tag{51d}$$

$$u_{y00} = 0 \qquad u_{x00} = 0 \tag{51e}$$

One finds that the fundamental dominant mode m = 0 and n = 0 in the duct with rigid walls given in Eqn. (51) is identical within the confined cross section of the duct with the acoustic uniform plane wave equations given for the infinite medium in Eqn. (23). Acoustic waves with every frequency can propagate in the duct in the fundamental mode (m = 0, n = 0) since for all frequencies $f > (f_c)_{00} = 0$. This acoustic wave of the fundamental mode will propagate in the duct with infinite medium velocity, wavelength and impedance, as if the duct of the rigid walls were not there. The $\theta = \frac{\pi}{2}$ in Eqn. (51b) indicates that the fundamental dominant mode propagates parallel to the duct walls with no reflections.

Let an acoustic uniform plane wave be propagating in an infinite fluid (air) in the z-direction. At each point in space it will have the

wave pressure scalar p, and the wave velocity vector $\bar{u}$ in the direction of propagation z. Both p and u_z will be constant along any phase plane $z = z_o$ at any particular time $t = t_o$, and will not vary in x or y directions. Since $u_x = 0$ and $u_y = 0$, one can introduce the four rigid walls of the duct of a rectangular cross-section directed in the z-direction without affecting the acoustic plane wave at all, since it does already obey the boundary conditions introduced by the rigid duct walls. Thus, the fundamental dominant mode of propagation of the acoustic wave in the duct with uniform rectangular cross section of rigid walls is that part of the same acoustic uniform plane wave in infinite medium which is confined by the duct walls without any changes in the pressure field, the velocity field, or any other parameters of propagation of the acoustic plane wave in an infinite fluid (air).

Assuming the particular case where the cross section of the duct is a square a = b, then one has the cut off wave number for mode (m,n) from Eqn. (50a) in the form:

$$(k_c)_{m,n} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{\pi}{a}\sqrt{m^2 + n^2} \quad (1/m) \quad a=b \qquad (52a)$$

where one has $(k_c)_{0,0} = 0$, $(k_c)_{1,0} = (k_c)_{0,1} = \frac{\pi}{a}$ and Eqn. (52a) may be rewritten in the form:

$$\frac{(k_c)_{m,n}}{(k_c)_{1,0}} = \sqrt{m^2 + n^2} \qquad \text{for } a=b \qquad (52b)$$

For the case of a square cross section a = b, the cut off frequency for mode (m,n) from Eqn. (50c) is in the form:

$$(f_c)_{m,n} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{a}\right)^2} = \frac{c}{2a}\sqrt{m^2 + n^2} \quad (1/\text{sec}) \quad a=b \tag{52c}$$

where one has $(f_c)_{0,0} = 0$, $(f_c)_{1,0} = (f_c)_{0,1} = \frac{c}{2a}$ and Eqn. (52c) may be rewritten in the form:

$$\frac{(f_c)_{m,n}}{(f_c)_{1,0}} = \sqrt{m^2 + n^2} \qquad \text{for } a=b \tag{52d}$$

The numerical values of Eqn. (52b) and Eqn. (52d) are given in the Table on the following pages.

If the signal frequency of the acoustic wave in the square duct $a = b$ is $f < (\frac{c}{2a})$, then only the fundamental dominant mode (0,0) can propagate and all the other modes will attenuate. If the signal frequency of the acoustic wave in the square duct $a = b$ is $(\frac{c}{2a}) < f < 1.4142(\frac{c}{2a})$ then only the acoustic modes (0,0),(1,0),(0,1) can propagate and all the other acoustic modes will attenuate. If the signal frequency of the acoustic wave in the square duct $a = b$ is $2(\frac{c}{2a}) < f < 2.2361(\frac{c}{2a})$, then only the acoustic modes (0,0),(1,0), (0,1),(1,1),(2,0),(0,2) will propagate and all the other acoustic modes will attenuate. If the signal frequency of the acoustic wave in the square duct $a = b$ is $f = 6.5(\frac{c}{2a})$ then the acoustic modes (5,4),(4,5) and all the modes listed above them will propagate, since $f > (f_c)_{m,n}$ for these modes, and the modes (6,3) (3,6) and all the modes listed below them in Table A will attenuate since $f < (f_c)_{m,n}$ for these modes.

TABLE A

CUT OFF FREQUENCIES FOR MODES (m,n) IN SQUARE DUCTS

mode (m,n) a = b				$\frac{(k_c)_{m,n}}{(k_c)_{1,0}} = \frac{(f_c)_{m,n}}{(f_c)_{1,0}} = \sqrt{m^2 + n^2}$
(0,0)				0
(1,0)	(0,1)			1.0000
(1,1)				1.4142
(2,0)	(0,2)			2.0000
(2,1)	(1,2)			2.2361
(2,2)				2.8284
(3,0)	(0,3)			3.0000
(3,1)	(1,3)			3.1623
(3,2)	(2,3)			3.6056
(4,0)	(0,4)			4.0000
(4,1)	(1,4)			4.1231
(3,3)				4.2426
(4,2)	(2,4)			4.4721
(4,3)	(3,4)	(5,0)	(0,5)	5.0000
(5,1)	(1,5)			5.0990
(5,2)	(2,5)			5.3852
(4,4)				5.6569
(5,3)	(3,5)			5.8310
(6,0)	(0,6)			6.0000
(6,1)	(1,6)			6.0828

TABLE A (continued)

mode (m,n)		a = b		$\frac{(k_c)_{m,n}}{(k_c)_{1,0}} = \frac{(f_c)_{m,n}}{(f_c)_{1,0}} = \sqrt{m^2 + n^2}$
(6,2)	(2,6)			6.3246
(5,4)	(4,5)			6.4031
(6,3)	(3,6)			6.7082
(7,0)	(0,7)			7.0000
(5,5)	(7,1)	(1,7)		7.0711
(6,4)	(4,6)			7.2111
(7,2)	(2,7)			7.2801
(7,3)	(3,7)			7.6158
(6,5)	(5,6)			7.8102
(8,0)	(0,8)			8.0000
(7,4)	(4,7)	(8,1)	(1,8)	8.0623
(8,2)	(2,8)			8.2462
(6,6)				8.4853
(8,3)	(3,8)			8.5440
(7,5)	(5,7)			8.6023
(8,4)	(4,8)			8.9443
(9,0)	(0,9)			9.0000
(9,1)	(1,9)			9.0554
(7,6)	(6,7)	(9,2)	(2,9)	9.2195
(8,5)	(5,8)			9.4340
(9,3)	(3,9)			9.4868
(9,4)	(4,9)			9.8489
(7,7)				9.8995
(8,6)	(6,8)	(10,0)	(0,10)	10.0000

CHAPTER V

VIBRATIONS OF THE PANEL - BASIC THEORY

In this chapter the equations governing the vibrations of a rectangular panel and its boundary conditions will be summarized as based on the theory of static and dynamic plates and the theory of elasticity. The detailed derivation of these equations may be found in two of the textbooks on the theory of plates listed in the Bibliography. In the first part the static plate will be considered and it will be extended later to dynamic analysis.

The shape of a plate is defined by describing the geometry of its middle surface, which bisects the plate thickness "h" at each point. The small deflection plate theory developed by Kirchhoff and Love is based on the following assumptions:

1. The material of the plate is elastic, homogeneous and isotropic.
2. The plate is initially flat (in the x-y plane).
3. $h \ll a$ and $h \ll b$ where "h" is the thickness of the plate (in the z-direction) and "a" and "b" are the dimensions of the plate (in the x-y plane).
4. $\eta \ll h$ where η is the lateral deflection or displacement of the plate (in the z-direction) and h is the thickness of the plate.
5. $\left|\frac{\partial \eta}{\partial x}\right| \ll 1, \left|\frac{\partial \eta}{\partial y}\right| \ll 1$ where $\frac{\partial \eta}{\partial x}$ is the slope of the deflected middle surface in the x-direction and $\frac{\partial \eta}{\partial y}$ is the slope of the deflected middle surface in the y-direction.
6. The deformations due to transverse shear forces will be neglected. Straight lines, initially normal to the middle surface (in the z-direction) remain the same after the deformation.

7. The deflection of the plate is produced by displacement η of points of the middle surface normal to its initial plane (in the z-direction).

8. The stresses normal to the middle surface are neglected.

9. The strains in the middle surface by inplane forces are neglected in comparison with the strains due to bending.

The validity of the Kirchhoff-Love plate theory is assumed for the small deflections of the panel due to the acoustic waves incident on it.

Taking the rectangular plate in the x-y plane, with the z axis perpendicular to it, the summation of all the forces in the z-direction on a small element of area of the plate is zero, and it yields the first equilibrium equation:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = -p_z \tag{53a}$$

where: p_z = the external load per unit area (or load density) (N/m^2) distributed on the upper surface of the plate.

q_x = the transverse lateral shearing force per unit of length of the y-direction (N/m).

q_y = the transverse lateral shearing force per unit of length of the x-direction (N/m).

Since the sum of all the moments acting on a small element of area around the y axis is zero, one obtains the second equilibrium equation:

$$\frac{\partial m_x}{\partial x} + \frac{\partial m_{yx}}{\partial y} = q_x \tag{53b}$$

where: m_x = the bending moment around the y axis per unit length of the y-direction (N)

m_{yx} = the torsional twisting moment around the y axis per unit length of the x-direction (N)

Since the sum of all the moments acting on a small element of area around the x axis is zero, one obtains the third equilibrium equation:

$$\frac{\partial m_y}{\partial y} + \frac{\partial m_{xy}}{\partial x} = q_y \qquad (53c)$$

where: m_y = the bending moment around the x axis per unit length of the x-direction (N)

m_{xy} = the torsional twisting moment around the x axis per unit length of the y-direction (N)

Since there are no forces in the x and y directions and no moments with respect to the z axis, the three Equations (53) completely define the equilibrium of the element. Substituting Eqn. (53b) and Eqn. (53c) in Eqn. (53a) and taking $m_{xy} = m_{yx}$, one obtains:

$$\frac{\partial^2 m_x}{\partial x^2} + 2\,\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_y}{\partial y^2} = -p_z \qquad (54)$$

By using Hooke's law one may find:

$$m_x = -D\left[\frac{\partial^2 \eta}{\partial x^2} + \nu\,\frac{\partial^2 \eta}{\partial y^2}\right] \qquad (55a)$$

$$m_y = -D\left[\frac{\partial^2 \eta}{\partial y^2} + \nu\,\frac{\partial^2 \eta}{\partial x^2}\right] \qquad (55b)$$

$$m_{xy} = m_{yx} = -(1 - \nu)\, D\,\frac{\partial^2 \eta}{\partial x \partial y} \qquad (55c)$$

where: $D = \dfrac{Eh^3}{12(1 - \nu^2)}$ = Bending or flexural rigidity of the plate (Nm).

E = Young's modulus of elasticity (N/m^2) which appears in Hooke's law.

ν = Poisson's ratio (ν = 0.3 for steel and aluminum).

h = thickness of the plate (m).

η = lateral deflection or displacement of the plate in the z-direction (m).

Substituting Eqn. (55) in Eqn. (54), one obtains:

$$\frac{\partial^4 \eta}{\partial x^4} + 2\frac{\partial^4 \eta}{\partial x^2 \partial y^2} + \frac{\partial^4 \eta}{\partial y^4} = \frac{p_z}{D} \tag{56a}$$

which may be also rewritten in the form:

$$D\nabla^4\eta = D\nabla^2\nabla^2\eta = D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 \eta = p_z \tag{56b}$$

Equation (56) was first obtained by Lagrange in 1811 and is called the Lagrange's equation of the non-homogeneous biharmonic equation. The static plate problem has been reduced to the solution of the linear partial differential equation (56) for the lateral deflection or displacement $\eta = \eta(x,y)$, subject to boundary conditions of the plate.

The transverse shearing forces q_x and q_y may be expressed in terms of the lateral deflections $\eta(x,y)$ by substituting Eqn. (55) in Eqn. (53b) and Eqn. (53c):

$$q_x = -D\frac{\partial}{\partial x}\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right] = -D\frac{\partial}{\partial x}\nabla^2\eta \tag{57a}$$

$$q_y = -D\frac{\partial}{\partial y}\left[\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right] = -D\frac{\partial}{\partial y}\nabla^2\eta \tag{57b}$$

Substituting Eqn. (57) in Eqn. (53), one obtains the differential equation (56).

The Lagrange's equation (56) is a fourth order partial differential equation for the lateral deflection or displacement $\eta = \eta(x,y)$. In order to obtain a unique solution, one should give two boundary conditions for each boundary or edge of the plate at $x = 0$, $x = a$, $y = 0$ and $y = b$, depending on the kind of support the edge has. Some different possible boundary conditions at the edge for the different cases are discussed in the following:

A. Simply supported edge - If the edge $x = a$ of the plate is simply supported, the deflection η along this edge must be zero. At the same time, this edge can rotate freely with respect to the edge line, i.e., there are no bending moments m_x along this edge. The boundary conditions for this edge are, using Eqn. (55a):

$$\eta = 0 \text{ and } m_x = -D\left[\frac{\partial^2 \eta}{\partial x^2} + \nu \frac{\partial^2 \eta}{\partial y^2}\right] = 0 \qquad \text{at } x = a \qquad (58a)$$

Since at $x = a$ one has $\eta = 0$, and therefore $\frac{\partial \eta}{\partial y} = 0$ and $\frac{\partial^2 \eta}{\partial y^2} = 0$ vanish as well, one may rewrite Eqn. (58a) alternatively:

$$\eta = 0 \text{ and } \frac{\partial^2 \eta}{\partial x^2} = 0 \qquad \text{at } x = a \qquad (58b)$$

Similarly, one has from Eqn. (55b) for the edge $y = b$:

$$\eta = 0 \text{ and } m_y = -D\left[\frac{\partial^2 \eta}{\partial y^2} + \nu \frac{\partial^2 \eta}{\partial x^2}\right] = 0 \qquad \text{at } y = b \qquad (58c)$$

or alternatively in the form:

$$\eta = 0 \text{ and } \frac{\partial^2 \eta}{\partial y^2} = 0 \qquad \text{at } y = b \qquad (58d)$$

Equations (58) represent the boundary conditions for a simply supported edge.

B. Clamped edge (also called fixed edge or built-in edge) - In this case the deflection along this edge is zero, and the tangent plane to the deflected plate along this edge coincides with the initial position of the plate. Therefore the boundary conditions for the clamped edges are given by:

$$\eta = 0, \ \frac{\partial \eta}{\partial x} = 0 \qquad \text{at } x = a \qquad (59a)$$

$$\eta = 0, \ \frac{\partial \eta}{\partial y} = 0 \qquad \text{at } y = b \qquad (59b)$$

C. Free edge (also called the statical boundary condition) - In this case the edge moment and the transverse shear force (v) are zero. The shearing force at the edge of the plate consists of the transverse shear and the effect of the torsional twisting moment. The boundary conditions at a free edge may be given in the form:

$$m_x = 0 \text{ and } v_x = q_x + \frac{\partial m_{xy}}{\partial y} = 0 \qquad \text{at } x = a \qquad (60a)$$

Substituting Eqns. (55a), (55c) and (57a), one obtains the boundary condition for the free edge in terms of lateral displacement η as follows:

$$m_x = -D[\frac{\partial^2 \eta}{\partial x^2} + \nu \frac{\partial^2 \eta}{\partial y^2}] = 0 \text{ and } v_x = -D[\frac{\partial^3 \eta}{\partial x^3} + (2-\nu)\frac{\partial^3 \eta}{\partial x \partial y^2}] = 0$$

$$\text{at } x = a \qquad (60b)$$

Similarly, one has for the edge at y = b:

$$m_y = 0 \text{ and } v_y = q_y + \frac{\partial m_{yx}}{\partial x} = 0 \qquad \text{at } y = b \quad (60c)$$

$$m_y = -D\left[\frac{\partial^2 \eta}{\partial y^2} + \nu \frac{\partial^2 \eta}{\partial x^2}\right] = 0 \text{ and } v_y = -D\left[\frac{\partial^3 \eta}{\partial y^3} + (2 - \nu)\frac{\partial^3 y}{\partial y \partial x^2}\right] = 0$$

$$\text{at } y = b \quad (60d)$$

Equations (60) represent the boundary conditions for a free edge.

In the dynamic analysis of the vibrations of the plate only the lateral motion in the z-direction is of interest, since the rotational inertia effects may be neglected. Therefore only the inertial forces, associated with the lateral translation of the plate in the z-direction will be considered. The partial differential equation governing the vibrations of the plate may be derived by applying the d'Alembert's dynamic equilibrium principle. The inertia force density associated with the lateral translation $\eta(x,y)$ of a plate element can be expressed by:

$$p_z^{\text{inertial}} = -\rho_p h \frac{\partial^2 \eta}{\partial t^2} \qquad (61)$$

where: p_z^{inertial} = the inertial force per unit area (N/m^2)

η = lateral displacement of the plate (m)

ρ_p = mass density of the plate material (Kg/m^3)

h = thickness of the plate (m)

$\rho_p h$ = mass density of the plate per unit area (Kg/m^2)

Extending the differential equation of static equilibrium (56b) by adding the inertial force Eqn. (61), one has:

$$D\nabla^4\eta = p_z + p_z^{\text{inertial}} = p_z - \rho_p h \frac{\partial^2\eta}{\partial t^2}$$

which can be rewritten in the form:

$$D\nabla^4\eta\ (x,y,t) + \rho_p h \frac{\partial^2\eta(x,y,t)}{\partial t^2} = p_z(x,y,t) \tag{62}$$

where Eqn. (62) represents the inhomogeneous partial differential equation of forced, undamped motion of the plate.

For the case of a freely vibrating plate, the external applied force $p_z(x,y,t) = 0$, and Eqn. (62) becomes:

$$D\nabla^4\eta(x,y,t) + \rho_p h \frac{\partial^2\eta(x,y,t)}{\partial t^2} = 0 \tag{63}$$

where Eqn. (63) represents the homogeneous partial differential equation of free, undamped flexural vibrations of the plate. Assuming harmonic time variation of the lateral displacement and of the external force per unit plate area:

$$\eta(x,y,t) = \text{Re}\sqrt{2}\ \underset{\sim}{\eta}(x,y)e^{-i\omega t} \tag{64a}$$

$$p_z(x,y,t) = \text{Re}\sqrt{2}\ \underset{\sim}{p}_z(x,y)e^{-i\omega t} \tag{64b}$$

and substituting in Eqn. (62) and Eqn. (63), one obtains:

$$D\nabla^4\underset{\sim}{\eta}(x,y) - \rho_p h\omega^2\ \underset{\sim}{\eta}(x,y) = \underset{\sim}{p}_z(x,y) \tag{65a}$$

$$D\nabla^4\underset{\sim}{\eta}(x,y) - \rho_p h\omega^2\ \underset{\sim}{\eta}(x,y) = 0 \tag{65b}$$

where $\underset{\sim}{\eta}(x,y)$ and $\underset{\sim}{p_z}(x,y)$ are complex phasors of the lateral displacement and the external force per unit plate area; the complex sign will be omitted when it is obvious.

The natural resonance frequencies of the free, undamped flexural vibrations of a simply supported rectangular plate of dimensions "a" and "b" may be found by assuming for the lateral displacement η of the homogeneous equation (63) the solution originally proposed by Navier in 1820 for the static plate:

$$\eta(x,y,t) = C\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-i\omega t} \tag{66a}$$

where m and n are integers. Eq. (64a) obeys the boundary conditions for a simply supported rectangular plate at all four edges, as given in Eqn. (58b) and Eqn. (58d). Substituting Eqn. (66a) in Eqn. (63), one obtains:

$$\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 - \frac{\rho_p h}{D}\omega^2 =$$

$$= \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 - \frac{\rho_p h}{D}\omega^2 = 0 \tag{66b}$$

where $\omega = \omega_{mn} = 2\pi f_{mn}$ is the resonance frequency of the mode (m,n) of the plate vibrations. From Eqn. (66b), one obtains the resonance frequencies of the simply supported rectangular plate as follows:

$$\omega_{mn} = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]\sqrt{\frac{D}{\rho_p h}} \quad \text{(1/sec)} \tag{66c}$$

$$f_{mn} = \frac{\pi}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sqrt{\frac{D}{\rho_p h}} \quad \text{(1/sec)} \tag{66d}$$

where: a,b = dimensions of the plate (m)

h = thickness of the plate (m)

ρ_p = mass density of the plate material (Kg/m^3)

D = bending or flexural rigidity of the plate (Nm)

The shape of the different modes of vibrations of the plate are given by Eqn. (66a) for the simply supported plate, while the corresponding resonance frequencies are given by Eqn. (66c) or Eqn. (66d). The fundamental mode of flexural vibration has a single sine wave in the x-direction and a single sine wave in the y-direction. The pertinent resonance frequency $f^R_{1,1}$ may be found by taking $m = 1$ and $n = 1$ in Eqn. (66d).

For a simply supported square plate where $a = b$ the mode ($m = 2$, $n = 1$) and the mode ($m = 1$, $n = 2$) have different shapes, but the same resonance frequency $f_{1,2} = f_{2,1}$; such modes of flexural vibrations are called degenerate modes. For a rectangular plate $a \neq b$ one has degenerate modes if the (a/b) ratio is a rational number.

Let us consider a simply supported rectangular plate of dimensions "a" and "b," acted upon by a given external arbitrary lateral load density $p_z(x,y)$, which varies harmonically in time with circular frequency ω. One has, therefore, using complex phasors:

$$p_z(x,y,t) = p_z(x,y)e^{-i\omega t} \tag{67a}$$

where $p_z(x,y)$ is a given function and can be expanded in a double Fourier series of sine terms as follows:

$$p_z(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} P_{m,n} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{67b}$$

$$P_{m,n} = \frac{4}{ab} \int_0^a \int_0^b p_z(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dxdy \qquad (67c)$$

In accordance with the method of Navier, the particular solution for the Lagrange equation in Eqn. (62) for the simply supported plate is given in the form, for the harmonic time varying case:

$$\eta(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{m,n} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-i\omega t} \qquad (68)$$

where $A_{m,n}$ are the unknown constants to be found for the particular solution. Substituting Eqn. (67) and Eqn. (68) in the Lagrange equation (62) and matching coefficients, one obtains:

$$A_{m,n}[D(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2})^2 - \rho_p h\omega^2] = P_{m,n} \qquad (69a)$$

where $A_{m,n}$ may be found from Eqn. (69a) while $P_{m,n}$ is calculated from Eqn. (67c). Substituting Eqn. (66c) in Eqn. (69a), one obtains:

$$\rho_p h\, A_{m,n}[\omega_{m,n}^2 - \omega^2] = P_{m,n} \qquad (69b)$$

or in the form:

$$A_{m,n} = \frac{P_{m,n}}{\rho_p h(\omega_{m,n}^2 - \omega^2)} \qquad (69c)$$

where $A_{m,n}$ in Eqn. (69c) is expressed in terms of the resonant frequency $\omega_{m,n}$ and the load frequency ω, to be substituted in Eqn. (68) for the final particular solution in the form of a Fourier series double sum.

The homogeneous solution of the homogeneous equation given in Eqn. (66a) should be added to the particular solution of the inhomogeneous equation given in Eqn. (68) and Eqn. (69), to give the general solution subject to given initial time conditions.

CHAPTER VI

ACOUSTIC PLANE WAVE INCIDENT ON AN INFINITE PANEL

In the present chapter the case of a plane acoustic wave in an infinite fluid (air), normally incident on an infinite panel (plate) will be discussed.

Assuming harmonic time variation $e^{-i\omega t}$, the inhomogeneous partial differential equation which governs the lateral displacement of the panel (plate) given in Eqn. (65a) can be rewritten as follows:

$$\nabla^4 \eta - \frac{\rho_p h \omega^2}{D} \eta = \frac{p_z}{D} \qquad (70a)$$

where: $\eta(x,y)$ = lateral displacement of the plate. (m)

$p_z(x,y)$ = external net force per unit area in the positive z-direction. (N/m^2)

ρ_p = mass density of the plate. (Kg/m^3)

h = thickness of the plate. (m)

$\omega = 2\pi f$ = circular frequency of the wave. (1/sec)

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ = bending or flexural rigidity of the plate. (Nm)

E = Young's modulus of elasticity. (N/m^2)

ν = Poisson's ratio (ν = 0.3 for steel and aluminum)

Equation (70a) may be rewritten in the form:

$$\nabla^4 \eta - \gamma^4 \eta = \frac{1}{D} p_z \qquad (70b)$$

where the plate wave number γ is defined by:

$$\gamma^4 = \frac{\rho_p h \omega^2}{D} = \frac{12(1-\nu^2)\rho_p \omega^2}{Eh^2} \qquad (1/m^4) \qquad (70c)$$

Assuming in Eqn. (70b) no external force density $p_z = 0$ and variation in only the y-direction $\eta = \eta(y)$, one will obtain:

$$\frac{d^4\eta}{dy^4} - \gamma^4\eta = 0 \qquad (71a)$$

which will be solved to give the following free motion elastic plate waves of the lateral displacement of the plate propagation along the plate:

$$\eta = \eta_o e^{\pm\gamma y - i\omega t} \quad \text{attenuation plate elastic waves} \qquad (71b)$$

$$\eta = \eta_o{}^{i(\pm\gamma y - \omega t)} \quad \text{propagating plate elastic waves} \qquad (71c)$$

where for the propagating wave Eqn. (71c) the plate wave number can be given in the form:

$$\gamma = \frac{\omega}{c_p} = \frac{2\pi f}{f\lambda_p} = \frac{2\pi}{\lambda_p} = [\frac{12(1-\nu^2)\rho_p}{E}]^{\frac{1}{4}}\sqrt{\frac{\omega}{h}} \qquad (1/m) \qquad (72a)$$

where: c_p = wave velocity of the elastic plate waves. (m/sec)

λ_p = wavelength of the elastic plate waves. (m)

f = frequency of the wave. (1/sec)

From Eqn. (72a), one obtains:

$$c_p = [\frac{E}{12(1-\nu^2)\rho_p}]^{\frac{1}{4}}\sqrt{h\omega} = [\frac{E}{12(1-\nu^2)\rho_p}]^{\frac{1}{4}}\sqrt{2\pi hf} \qquad (m/sec) \qquad (72b)$$

$$\lambda_p = \left[\frac{E}{12(1-\nu^2)\rho_p}\right]^{\frac{1}{4}} 2\pi\sqrt{\frac{h}{\omega}} = \left[\frac{E}{12(1-\nu^2)\rho_p}\right]^{\frac{1}{4}} \sqrt{\frac{2\pi h}{f}} \quad (m) \qquad (72c)$$

where $c_p = f\lambda_p$ and $c_p = c_p(f)$. The plate is dispersive medium for the transverse displacement waves propagating along the plate.

Let an infinite panel (plate) be situated at z = 0 in the x-y plane. Let an acoustic plane wave propagating in the positive z-direction be normally incident on the infinite plate and be given by Eqns. (23) in the form:

$$p_i = P_I e^{i(kz-\omega t)}; \quad u_{zi} = \frac{p_i}{\rho c} \qquad z \leq 0 \qquad (73a)$$

This incident acoustic plane wave will be reflected by the infinite plate in the form of a reflected acoustic plane wave propagating in the negative z-direction and given by Eqns. (24) in the form:

$$p_r = P_R e^{-i(kz+\omega t)}; \quad u_{zr} = -\frac{p_r}{\rho c} \qquad z \leq 0 \qquad (73b)$$

The incident acoustic plane wave will cause the infinite plate to vibrate harmonically in the lateral positive z-direction in the form:

$$\eta = Ae^{-i\omega t} \qquad z = 0 \qquad (74a)$$

where η is the lateral displacement of the plate in the positive z-direction; the velocity of the plate in the positive z-direction may be found from Eqn. (74a) to give:

$$u_{zp} = \frac{d\eta}{dt} = -i\omega Ae^{-i\omega t} \qquad z = 0 \qquad (74b)$$

No plate boundary conditions are applied to Eqn. (74a) since the plate is of infinite dimensions in both x- and y-directions.

The vibrations of the plate given in Eqn. (74) will generate on the other side of the plate, $z > 0$, an acoustic transmitted plane wave in the positive z-direction which will be given by Eqns. (23) in the form:

$$p_t = P_T e^{i(kz-\omega t)}; \quad u_{zt} = \frac{p_t}{\rho c} \qquad z \geq 0 \qquad (75)$$

The total external force per unit area p_z on the plate at $z = 0$ in the positive z-direction by the incident acoustic wave Eqn.(73a), the reflected acoustic wave Eqn. (73b) and the transmitted acoustic wave Eqn. (75), is given by:

$$p_z = P_I e^{-i\omega t} + P_R e^{-i\omega t} - P_T e^{-i\omega t} \qquad (76a)$$

where the incident and reflected acoustic waves at $z < 0$ pressure the plate in the positive z-direction and the transmitted acoustic wave at $z > 0$ pressures the plate in the negative z-direction. Substituting Eqn. (74a) and Eqn. (76a) in Eqn. (70b), one finds $\nabla^4\eta = 0$ and it becomes:

$$-\gamma^4 A = \frac{1}{D}(P_I + P_R - P_T) \qquad (76b)$$

where the factor $e^{-i\omega t}$ has been cancelled. Since $\gamma^4 D = \rho_p h\omega^2$, one may rewrite Eqn. (76b) in the form:

$$-\rho_p h\omega^2 A = P_I + P_R - P_T \tag{76c}$$

The plate velocity u_{zp} and the fluid (air) velocity of the acoustic waves on either side of the plate should be identical. On the positive side of the plate $z > 0$ at the plate $z = 0$, denoted by $z = 0_+$, the plate velocity u_{zp} should be identical with the transmitted acoustic wave velocity vector n_{zt} as follows:

$$u_{zp}(z = 0_+) = u_{zt}(z = 0_+) \tag{77a}$$

Substituting Eqn. (74b) and Eqn. (75) in Eqn. (77a), one obtains:

$$-i\omega A = \frac{1}{\rho c} P_T \tag{77b}$$

On the negative side of the plate $z < 0$ at the plate $z = 0$, denoted by $z = 0_-$, the plate velocity u_{zp} should be identical with the sum of the incident acoustic wave velocity vector u_{zi} and the reflected acoustic wave velocity vector u_{zr}, as follows:

$$u_{zp}(z = 0_-) = u_{zi}(z = 0_-) + u_{zr}(z = 0_-) \tag{78a}$$

Substituting Eqns. (73a), (73b) and (74b) in Eqn. (78a), one obtains:

$$-i\omega A = \frac{1}{\rho c} P_I - \frac{1}{\rho c} P_R = \frac{1}{\rho c} (P_I - P_R) \tag{78b}$$

From Eqn. (77b) and Eqn. (78b), one obtains:

$$P_I = P_R + P_T \tag{79}$$

The pressure of the incident wave is divided among the pressure of the reflected wave and the pressure of the transmitted wave.

From a given acoustic incident plane wave P_I, on the infinite plate, one may obtain the unknown P_R, P_T and A by solving the three linear inhomogeneous equations (76c), (77b) and (78b).

From Eqn. (77b), one obtains:

$$P_T = -i\omega\rho c\ A \tag{80a}$$

By substituting the additional equation (79) in Eqn. (76c), one obtains:

$$P_R = -\frac{\rho_p h\omega^2}{2} A \tag{80b}$$

Substituting Eqns. (80) in Eqn. (79), one obtains:

$$P_I = (-i\omega\rho c - \frac{\rho_p h\omega^2}{2})A = -i\omega\rho c(1 - i\,\frac{\rho_p h\omega}{2\rho c})A \tag{80c}$$

The coupling parameter μ between the plate and the air is defined as follows:

$$\mu = \frac{2\rho}{\rho_p h} \qquad (1/m)$$

where: ρ = air density. (Kg/m^3)

ρ_p = plate material density. (Kg/m^3)

h = thickness of plate. (m)

By substituting the parameter $\mu = \frac{2\rho}{\rho_p h}$ and the acoustic wave number $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ in Eqn. (80c), one obtains:

$$P_I = -i\omega\rho c \left(1 - i \frac{k}{\mu}\right) A \qquad (80d)$$

where $(\frac{k}{\mu})$ is a non-dimensional number. From Eqn. (79) and Eqn. (80), one obtains:

$$A = \frac{\frac{i}{\omega\rho c} P_I}{1 - i\frac{k}{\mu}} \qquad (81a)$$

$$P_T = \frac{P_I}{1 - i\frac{k}{\mu}} \qquad (81b)$$

$$P_R = \frac{-i\frac{k}{\mu} P_I}{1 - i\frac{k}{\mu}} \qquad (81c)$$

The Transmission Loss coefficient is defined as the ratio of the incident acoustic wave pressure power to the transmitted acoustic wave pressure power. From Eqns. (26), (73a) and (75), one obtains the Transmission Loss coefficient in decibels in the form:

$$TL_{db} = 10 \log \frac{|p_i|^2}{|p_t|^2} = 10 \log \left|\frac{P_I}{P_T}\right|^2 \qquad (82a)$$

where TL_{db} is the Transmission Loss coefficient expressed in decibels. Substituting Eqn. (81b) for the present case in Eqn. (82a), one has:

$$TL_{db} = 10 \log \left|1 - i \frac{k}{\mu}\right|^2 = 10 \log \left[1 + \left(\frac{k}{\mu}\right)^2\right] =$$

$$= 10 \log \left[1 + \left(\frac{\omega\rho_p h}{2\rho c}\right)^2\right] \qquad (82b)$$

Usually the measurements are done by two microphones which measure the pressure power of the acoustic waves. One microphone is situated on the source side of the panel at $z = -d_1$ (the source microphone) and it measures the total acoustic wave pressure power at that point, both the incident and the reflected acoustic waves. The other microphone is situated on the other side of the panel at $z = +d_2$ (the receiver microphone) and it measures the total acoustic wave pressure power at that point of the transmitted acoustic wave. The Noise Reduction coefficient is defined as the ratio of the total acoustic wave pressure power measured by the source microphone at $z = -d_1$ to the acoustic wave pressure power, measured by the receiver microphone at $z = +d_2$. From Eqns. (73) and (75), one obtains the Noise Reduction coefficient in decibels in the form:

$$NR_{db} = 10 \log \frac{|p_i + p_r|^2_{z=-d_1}}{|p_t|^2_{z=+d_2}} = 10 \log \left| \frac{P_I e^{-ikd_1} + P_R e^{+ikd_1}}{P_T e^{ikd_2}} \right|^2 \qquad (83a)$$

where NR_{db} is the Noise Reduction coefficient expressed in decibels. Substituting Eqn. (81b) and Eqn. (81c) for the present case in Eqn. (83a) one obtains:

$$NR_{db} = 10 \log \left| \frac{e^{-ikd_1} - \frac{i\frac{k}{\mu}}{1-i\frac{k}{\mu}} e^{+ikd_1}}{\frac{1}{1-i\frac{k}{\mu}}} \right|^2 = 10 \log \left| (1-i\frac{k}{\mu}) e^{-ikd_1} - i\frac{k}{\mu} e^{+ikd_1} \right|^2 =$$

$$= 10 \log \left| e^{-ikd_1} - i\frac{k}{\mu}\, 2 \cos kd_1 \right|^2 =$$

$$= 10 \log \left| \cos kd_1 - i \sin kd_1 - i \frac{2k}{\mu} \cos kd_1 \right|^2 =$$

$$= 10 \log [\cos^2 kd_1 + (\sin kd_1 + \frac{2k}{\mu} \cos k d_1)^2] =$$

$$= 10 \log [1 + \frac{4k}{\mu} \sin kd_1 \cos kd_1 + (\frac{2k}{\mu})^2 \cos^2 kd_1] =$$

$$= 10 \log [1 + \frac{2k}{\mu} \sin 2 kd_1 + (\frac{2k}{\mu})^2 \cos^2 kd_1] \tag{83b}$$

where the Noise Reduction coefficient NR_{db} depends on the position of the source microphone $z = -d_1$. For the particular case $kd_1 = 2\pi \frac{d_1}{\lambda} << 1$, Eqn. (83b) will reduce to:

$$NR_{db} = 10 \log [1 + (\frac{2k}{\mu})^2] = 10 \log[1 + (\frac{\omega\rho_p h}{\rho c})^2]$$

$$\text{for } |kd_1| << 1 \tag{83c}$$

for the case of $kd_1 << 1$ the Noise Reduction coefficient NR_{db} does not depend on the position of the source microphone $z = -d_1$.

The Insertion Loss coefficient is defined as the ratio of the incident acoustic wave pressure power measured by the receiver microphone without the panel, to the transmitted acoustic wave pressure power measured at the same point by the receiver microphone with the panel installed. From Eqns. (73a) and (75), one obtains the Insertion Loss coefficient in decibels for the present case in the form:

$$IL_{db} = 10 \log \frac{|p_i|^2}{|p_t|^2} = 10 \log \left|\frac{P_I}{P_T}\right|^2 \tag{84a}$$

where IL_{db} is the Insertion Loss coefficient expressed in decibels.

Substituting Eqn. (81b) for the present case in Eqn. (84a), one has:

$$IL_{db} = 10 \log \left|1 - i \frac{k}{\mu}\right|^2 = 10 \log[1 + (\frac{k}{\mu})^2] = 10 \log[1 + (\frac{\omega\rho_p h}{2\rho c})^2] \quad (84b)$$

From Eqns. (82b) and (84b), one finds for the present case:

$$TL_{db} = IL_{db} \quad (85a)$$

From Eqns. (82b) and (83c), one finds that for $(k/\mu) >> 1$ one has:

$$NR_{db} - TL_{db} = 10 \log(\frac{2k}{\mu})^2 - 10 \log(\frac{k}{\mu})^2 = 10 \log \frac{(\frac{2k}{\mu})^2}{(\frac{k}{\mu})^2} =$$

$$= 10 \log(2^2) = 20 \log 2 = 20 \times 0.3010 \cong 6 \text{ db}$$

which may be rewritten in the form:

$$NR_{db} = TL_{db} + 6db = IL_{db} + 6db \text{ for } k >> \mu \text{ and } |kd_1| << 1 \quad (85b)$$

Equation (82b) is generally known as the "normal incidence mass law." For the case $k >> \mu$ Eqn. (82b) becomes:

$$TL_{db} = 20 \log (\frac{k}{\mu}) = 20 \log (\frac{\omega\rho_p h}{2\rho c}) = 20 \log (\frac{\pi\rho_p hf}{\rho c}) \text{ for } k>>\mu \quad (86a)$$

From Eqn. (86a), one sees that when the plate surface density ($\rho_p h$) is doubled, the Transmission Loss coefficient increases by 6 db. One also sees that if the frequency f doubles (= the frequency is raised by

one octave) the Transmission Loss coefficient increases by 6 db. Eqn. (86a) may be rewritten in the form:

$$TL_{db} = 20 \log \left(\frac{\pi \rho_p h}{\rho c}\right) + 20 \log f = C + 20 \log f \quad \text{for } k >> \mu \qquad (86b)$$

Thus if the Transmission Loss coefficient in decibels is drawn against (log f), it will give a straight line, with a slope of 20 (= doubling the frequency will increase TL_{db} by 6 db) and it will have the value $TL_{db} = C$ at the frequency f = 1 (1/sec), or the value $TL_{db} = (C+40)$ at the frequency f = 100 (1/sec). In accordance with Eqn. (85b) the Noise Reduction coefficients NR_{db} for $k >> \mu$ will be a straight line parallel to the TL_{db} straight line, and 6 db above it. For the case $k << \mu$ one finds from Eqns. (82b) and (83c) that both the TL_{db} coefficient and the NR_{db} coefficient should approach the value zero, and the normally incident acoustic plane wave for very small frequencies will propagate through the infinite panel as if it was completely transparent to it.

CHAPTER VII

ACOUSTIC PLANE WAVE INCIDENT ON A FINITE PANEL

In the present chapter the case of a plane acoustic wave normally incident on a finite clamped panel (plate) in a rigid duct will be discussed.

It has been shown before that the fundamental mode of propagation of the acoustic waves in a rigid duct are identical with the plane acoustic wave propagating in the infinite fluid (air) limited by the walls of the rigid duct. The introduction of the walls of the rigid duct parallel to the direction of propagation does not affect the plane acoustic wave, since the boundary conditions of the walls of the rigid duct are obeyed; the fundamental mode of propagation of the acoustic wave in the duct propagates in the duct as a plane acoustic wave, as if the duct rigid walls were not there. Thus, the introduction of the duct rigid walls parallel to the direction of propagation does not alter the acoustic waves discussed in the previous chapter.

When one introduces a finite rectangular clamped panel (plate) in the duct, one should apply the boundary conditions for a clamped rectangular plate as discussed previously. One is not able to use the solution for the infinite panel (plate) discussed in the previous chapter without any boundary conditions. Thus, the introduction of the duct rigid walls parallel to the direction of propagation does not affect the plane acoustic waves discussed in the previous chapter, but it does introduce the effect of the clamped boundary conditions of the panel (plate), which is now clamped to the rigid wall of the duct. The solution given in the previous chapter for the infinite panel (plate)

for the lateral displacement η in Eqns. (74a) and (81a) do not obey the boundary conditions for the clamped edges of the panel (plate) at $x = 0$, $x = a$, $y = 0$, $y = b$ as given in Eqn. (59). Therefore the solution for the infinite panel (plate) given in the previous chapter should be altered.

In order to simplify the analysis and limit it only to the fundamental resonance frequency of the panel (plate) in the present chapter, it is assumed that the effect of the four clamped edges of the plate is equivalent to a spring, forcing the plate to return to its position of equilibrium without acoustic plane wave incident on it. Thus, the plate has been idealized by introducing the effect of the clamped edge boundary condition as an equivalent system of a single degree of freedom only. For small lateral displacements of the plate η, the spring-like force per unit area is proportional to the lateral displacement of the plate and can be expressed in the following form by using the stiffness constant of the plate:

$$p_z^{\text{spring}} = - K\eta \tag{87}$$

where: p_z^{spring} = spring like force density. (N/m^2)

η = the lateral displacement of the plate. (m)

K = the stiffness constant of the plate. (N/m^3)

The formulation of the structural damping of the vibrating plate can be accomplished by expressing the damping force per unit area as a purely imaginary constant, being proportional to the small lateral displacement, as follows, where $i = \sqrt{-1}$.

$$p_z^{damping} = -i\alpha K\eta \qquad (88)$$

where: $p_z^{damping}$ = the damping force density. (N/m^2)

η = the lateral displacement of the plate. (m)

K = the stiffness constant of the plate. (N/m^3)

α = the damping factor. (non dimensional)

Equation (88) has been employed extensively in the dynamic analysis of aerospace structures.

The inertia force density associated with the lateral translation $\eta(x,y)$ of a plate element has been given in Eqn. (61) as follows:

$$p_z^{inertial} = -\rho_p h \frac{d^2\eta}{dt^2} \qquad (89)$$

where: $p_z^{inertial}$ = the inertial force per unit area. (N/m^2)

ρ_p = mass density of the plate material. (Kg/m^3)

h = thickness of the plate. (m)

$\rho_p h$ = mass of plate per unit area. (Kg/m^2)

Using Eqns. (87) and (89), the natural fundamental circular resonance frequency ω_o will be defined for a spring-like system of one degree of freedom of this type as follows:

$$\omega_o = \sqrt{\frac{K}{\rho_p h}} \qquad f_o = \frac{1}{2\pi}\sqrt{\frac{K}{\rho_p h}} \qquad (1/sec) \qquad (90)$$

Extending the differential equation of static equilibrium, Eqn.(56b), by adding the spring force, Eqn. (87), the damping force, Eqn. (88), and the inertial force, Eqn. (89), one obtains:

$$D\nabla^4\eta = p_z + p_z^{\text{spring}} + p_z^{\text{damping}} + p_z^{\text{inertial}} =$$

$$= p_z - K\eta - i\alpha K\eta - \rho_p h \frac{\partial^2\eta}{\partial t^2}$$

which can be rewritten in the form:

$$D\nabla^4\eta + \rho_p h \frac{\partial^2\eta}{\partial t^2} + K(1 + i\alpha)\eta = p_z(x,y,t) \tag{91a}$$

Substituting from Eqn. (90) $K = \rho_p h\omega_o^2$ in Eqn. (91a), one has:

$$D\nabla^4\eta + \rho_p h[\frac{\partial^2\eta}{\partial t^2} + \omega_o^2(1 + i\alpha)\eta] = p_z(x,y,t) \tag{91b}$$

where Eqns.(91) represent the inhomogeneous partial differential equation of forced and damped motion of the clamped plate, taking only the fundamental circular resonance frequency ω_o into account.

Assuming harmonic time variation $e^{-i\omega t}$ as in Eqn. (64), one will obtain from Eqn. (91b):

$$D\nabla^4\eta - \rho_p h\omega^2[1 - \frac{\omega_o^2}{\omega^2}(1 + i\alpha)]\eta = p_z(x,y) \tag{92a}$$

which may be rewritten in the form:

$$\nabla^4\eta - \gamma_o^4\eta = \frac{1}{D}p_z \tag{92b}$$

where the plate wave number γ_o for the present case is defined by:

$$\gamma_o^4 = \frac{\rho_p h\omega^2}{D}[1 - \frac{\omega_o^2}{\omega^2}(1 + i\alpha)] = \gamma^4[1 - \frac{\omega_o^2}{\omega^2}(1 + i\alpha)] \quad (1/m^4) \tag{92c}$$

where: $$\gamma^4 = \frac{\rho_p h\omega^2}{D} = \frac{12(1-\nu^2)\rho_p\omega^2}{Eh^2} \qquad (1/m^4)$$

ω = circular frequency of the external force. (1/sec)

ω_o = fundamental circular resonance frequency of the plate. (1/sec)

α = the damping factor. (non-dimensional)

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ = bending or flexural rigidity of the plate. (Nm)

For the particular case K = 0, one has $\omega_o = 0$ and $\gamma_o = \gamma$.

Let the clamped panel (plate) be situated at z = 0 in the x-y plane in the rigid duct. Let the fundamental or dominant mode (m = 0, n = 0) of the acoustic wave propagating in the positive z-direction in the rigid duct be normally incident on the clamped plate and be given in the form:

$$p_i = P_I e^{i(kz-\omega t)}; \quad u_{zi} = \frac{p_i}{\rho c} \qquad z < 0 \qquad (93a)$$

The reflected fundamental mode from the clamped plate will propagate in the negative z-direction and is given by:

$$p_r = P_R e^{-i(kz+\omega t)}; \quad u_{zr} = -\frac{p_r}{\rho c} \qquad z < 0 \qquad (93b)$$

The incident acoustic wave will cause the clamped plate to vibrate harmonically in the lateral positive z-direction in the form:

$$\eta = Ae^{-i\omega t}; \quad u_{zp} = \frac{d\eta}{dt} = -i\omega Ae^{-i\omega t} \qquad z = 0 \qquad (93c)$$

The vibrations of the plate in Eqn. (93c) will generate on the other side of the plate $z > 0$ the fundamental mode of the acoustic transmitted wave in the positive z-direction in the form:

$$p_t = P_T e^{i(kz-\omega t)}; \quad u_{zt} = \frac{p_t}{\rho c} \qquad z > 0 \qquad (93d)$$

Substituting Eqns. (93) in Eqn. (92b), one finds similarly to Eqn. (76b):

$$-\gamma_o{}^4 A = \frac{1}{D}(P_I + P_R - P_T) \qquad (94a)$$

Similarly to Eqn. (77b) and Eqn. (78b), one also finds from Eqns. (93):

$$-i\omega A = \frac{1}{\rho c} P_T \qquad (94b)$$

$$-i\omega A = \frac{1}{\rho c} (P_I - P_R) \qquad (94c)$$

From Eqns. (94b) and (94c), one also obtains:

$$P_I = P_R + P_T \qquad (95)$$

From Eqn. (94b), one obtains:

$$P_T = -i\omega\rho c A \qquad (96a)$$

Substituting Eqn. (95) in Eqn. (94a), one obtains:

$$P_R = -\frac{D\gamma_o^4}{2} \cdot A \tag{96b}$$

Substituting Eqns. (96) in Eqn. (95), one obtains:

$$P_I = (-i\omega\rho c - \frac{D\gamma_o^4}{2})A = -i\omega\rho c(1 - i\frac{D\gamma_o^4}{2\omega\rho c})A \tag{96c}$$

From Eqns. (96), one obtains:

$$A = \frac{\frac{i}{\omega\rho c} P_I}{1 - i\frac{D\gamma_o^4}{2\omega\rho c}} \tag{97a}$$

$$P_T = \frac{P_I}{1 - i\frac{D\gamma_o^4}{2\omega\rho c}} \tag{97b}$$

$$P_R = \frac{-i\frac{D\gamma_o^4}{2\omega\rho c} P_I}{1 - i\frac{D\gamma_o^4}{2\omega\rho c}} \tag{97c}$$

The Transmission Loss coefficient defined in Eqn. (82a) may be found from Eqn. (97b) to give for the present case:

$$TL_{db} = 10 \log \left|\frac{P_I}{P_T}\right|^2 = 10 \log \left|1 - i\frac{D\gamma_o^4}{2\omega\rho c}\right|^2 \tag{98a}$$

Substituting Eqn. (92c) in Eqn. (98a) and defining $k = \frac{\omega}{c}$ and $\mu = \frac{2\rho}{\rho_p h}$ one has:

$$TL_{db} = 10 \log \left| 1 - i \frac{k}{\mu}[1 - \frac{\omega_o^2}{\omega^2}(1 + i\alpha)] \right|^2 =$$

$$= 10 \log \left| (1 - \frac{k}{\mu} \frac{\omega_o^2 \alpha}{\omega^2}) - i \frac{k}{\mu} (1 - \frac{\omega_o^2}{\omega^2}) \right|^2 =$$

$$= 10 \log [(1 - \frac{k}{\mu} \frac{\omega_o^2 \alpha}{\omega^2})^2 + (\frac{k}{\mu})^2 (1 - \frac{\omega_o^2}{\omega^2})^2] \qquad (98b)$$

For the particular case of an infinite plate $K = 0$, therefore $\omega_o = 0$, and Eqn. (98b) becomes identical with Eqn. (82b).

The Noise Reduction coefficient defined in Eqn. (83a) may be found from Eqn. (97) to give for the present case:

$$NR_{db} = 10 \log \frac{|p_i + p_r|^2_{z=-d_1}}{|p_t|^2_{z=+d_2}} = 10 \log \left| \frac{P_I e^{-ikd_1} + P_R e^{+ikd_1}}{P_T e^{ikd_2}} \right|^2 =$$

$$= 10 \log \left| \frac{e^{-ikd_1} - \dfrac{i \frac{D\gamma_o^4}{2\omega\rho c}}{1 - i \frac{D\gamma_o^4}{2\omega\rho c}} e^{+ikd_1}}{\dfrac{1}{1 - i \frac{D\gamma_o^4}{2\omega\rho c}}} \right|^2 =$$

$$= 10 \log \left| (1 - i \frac{D\gamma_o^4}{2\omega\rho c}) e^{-ikd_1} - i \frac{D\gamma_o^4}{2\omega\rho c} e^{+ikd_1} \right|^2$$

$$= 10 \log \left| e^{-ikd_1} - i \frac{D\gamma_o^4}{2\omega\rho c} 2 \cos kd_1 \right|^2 \qquad (99a)$$

Substituting Eqn. (92c) in Eqn. (99a) and defining $k = \frac{\omega}{c}$ and $\mu = \frac{2\rho}{\rho_p h}$ Eqn. (99a) may be developed as follows:

$$NR_{db} = 10 \log \left| e^{-ikd_1} - i \frac{2k}{\mu} [1 - \frac{\omega_o^2}{\omega^2} (1 + i\alpha)] \cos kd_1 \right|^2 =$$

$$= 10 \log \left| \cos kd_1 - i \sin kd_1 - \frac{2k}{\mu} \cos kd_1 [i(1 - \frac{\omega_o^2}{\omega^2}) + \frac{\omega_o^2 \alpha}{\omega^2}] \right|^2 =$$

$$= 10 \log \left| \cos kd_1 [1 - \frac{2k}{\mu} \frac{\omega_o^2 \alpha}{\omega^2}] - i[\sin kd_1 + \frac{2k}{\mu} (1 - \frac{\omega_o^2}{\omega^2}) \cos kd_1] \right|^2 =$$

$$= 10 \log \left\{ \cos^2 kd_1 [1 - \frac{2k}{\mu} \frac{\omega_o^2 \alpha}{\omega^2}]^2 + [\sin kd_1 + \frac{2k}{\mu} (1 - \frac{\omega_o^2}{\omega^2}) \cos kd_1]^2 \right\} \quad (99b)$$

For the particular case of an infinite plate $K = 0$, therefore $\omega_o = 0$, Eqn. (99b) becomes identical with Eqn. (83b).

For the particular case that $kd_1 = 2\pi \frac{d_1}{\lambda} << 1$, one has $\cos kd_1 \cong 1$ and $\sin kd_1 \cong 0$ and Eqn. (99b) becomes:

$$NR_{db} = 10 \log[(1 - \frac{2k}{\mu} \frac{\omega_o^2 \alpha}{\omega^2})^2 + (\frac{2k}{\mu})^2 (1 - \frac{\omega_o^2}{\omega^2})^2]$$

$$\text{where } |kd_1| << 1 \qquad (99c)$$

For the particular case of an infinite plate $K = 0$, therefore $\omega_o = 0$, Eqn. (99c) becomes identical with Eqn. (83c). For the case of $|kd_1| << 1$ given in Eqn. (99c), the Noise Reduction coefficient NR_{db} does not depend on the position of the source microphone $z = -d_1$.

The Insertion Loss coefficient in decibels defined in Eqn. (84a) may be found from Eqn. (97b) to give for the present case as in Eqn. (98b):

$$IL_{db} = 10 \log \left|\frac{p_i}{p_t}\right|^2 = 10 \log \left|\frac{P_I}{P_T}\right|^2 = 10 \log \left|1 - i\,\frac{D\gamma_o^4}{2\omega\rho c}\right|^2 =$$

$$= 10 \log\left[\left(1 - \frac{k}{\mu}\frac{\omega_o^2\alpha}{\omega^2}\right)^2 + \left(\frac{k}{\mu}\right)^2\left(1 - \frac{\omega_o^2}{\omega^2}\right)^2\right] \qquad (100)$$

where IL_{db} is the Insertion Loss coefficient expressed in decibels.

From Eqn. (98) and Eqn. (100), one finds for the present case:

$$TL_{db} = IL_{db} \qquad (101a)$$

From Eqn. (98b) and Eqn. (99c), one finds that for:

$$\left|\frac{k}{\mu}\left(1 - \frac{\omega_o^2}{\omega^2}\right)\right| >> \left|1 - \frac{k}{\mu}\frac{\omega_o^2\alpha}{\omega^2}\right| \quad \text{one has:}$$

$$NR_{db} - TL_{db} = 10 \log 4 = 20 \log 2 \cong 6 \text{ db}$$

which may be rewritten in the form:

$$NR_{db} = TL_{db} + 6 \text{ db} = IL_{db} + 6 \text{ db} \qquad (101b)$$

Assuming that in the region of frequencies ω considered in this report $|kd_1| << 1$ and substituting $k = \frac{\omega}{c}$ in Eqn. (99c), one obtains:

$$NR_{db} = 10 \log \left[\left(1 - \frac{2}{\mu c}\frac{\omega_o^2\alpha}{\omega}\right)^2 + \left(\frac{2}{\mu c}\right)^2\left(\frac{\omega^2 - \omega_o^2}{\omega}\right)^2\right] \qquad (102a)$$

Since $\omega = 2\pi f$ and $\omega_o = 2\pi f_o$, one may rewrite Eqn. (102a) in the form:

$$NR_{db} = 10 \log \left[\left(1 - \frac{4\pi}{\mu c}\frac{f_o^2\alpha}{f}\right)^2 + \left(\frac{4\pi}{\mu c}\right)^2\left(\frac{f^2 - f_o^2}{f}\right)^2\right] \qquad (102b)$$

where: NR_{db} = Noise Reduction coefficient in decibels.

ω = the circular frequency of the acoustic wave. (1/sec)

f = the frequency of the acoustic wave. (1/sec)

ω_o = the circular fundamental resonance frequency of the plate. (1/sec)

f_o = the fundamental resonance frequency of the plate. (1/sec)

c = the velocity of the plane acoustic wave in infinite medium. (m/sec)

α = the damping factor of the vibrating plate. (non dimensional)

$\mu = \frac{2\rho}{\rho_p h}$ = the coupling coefficient μ. (1/m)

The Noise Reduction curve in Eqn. (102b) as a function of frequency f can be divided into three major regions:

A. <u>The mass controlled region</u>: This is the region where the frequency of the acoustic wave f is much larger than the fundamental resonance frequency of the plate f_o such that $f^2 >> f_o^2$. In this region Eqn. (102b) becomes:

$$NR_{db} = 10 \log [(\frac{4\pi}{\mu c})^2 f^2] = 20 \log [\frac{4\rho}{\mu c} f] =$$

$$= 20 \log (\frac{4\pi}{\mu c}) + 20 \log f$$

$$= 20 \log (\frac{2\pi\rho_p h}{\rho c}) + 20 \log f$$

$$= C + 20 \log f \qquad \text{for } f^2 >> f_o^2 \qquad (103)$$

In this region doubling the mass per unit area of the plate ($\rho_p h$) will increase NR_{db} by 6 decibels, and raising the frequency f of the acoustic wave by one octave (= doubling the frequency f) will increase NR_{db} by 6 decibels. Drawing $NR_{db}(\log f)$ on a log-log paper will give a straight line with a slope of 20 (increase of NR_{db} by 6 db for every octave of f) and a constant C such that:

$$NR_{db}(f = 1) = C = 20 \log\left(\frac{2\pi\rho_p h}{\rho c}\right) = -20 \log \left(\frac{\rho c}{2\pi\rho_p h}\right)$$

$$NR_{db}(f = 100) = C + 40 \text{ db}$$

B. <u>The damping controlled region</u>: This is the region where the frequency of the acoustic wave f is approximately the same as the fundamental resonance frequency of the plate f_o such that $f \simeq f_o$. Since $|\alpha| << 1$ and the first term in Eqn. (102b) varies slowly with f as compared to the second term, Eqn. (102b) for this region will become:

$$NR_{db} = 10 \log \left[\left(1 - \frac{4\pi}{\mu c} f_o\alpha\right)^2 + \left(\frac{4\pi}{\mu c}\right)^2\left(\frac{f^2 - f_o^{\,2}}{f}\right)^2\right]$$

$$\text{for } f \simeq f_o \tag{104a}$$

when $f = f_o$, Eqn. (104a) becomes:

$$NR_{db} = 10 \log \left(1 - \frac{4\pi}{\mu c} f_o\alpha\right)^2 = 20 \log \left(1 - \frac{4\pi}{\mu c} f_o\alpha\right) =$$

$$= 20 \log \left(1 - \frac{2\pi\rho_p h}{\rho c} f_o\alpha\right) \qquad \text{for } f = f_o \tag{104b}$$

For the case in Eqn. (104b) where all the terms are positive including $f_o > 0$ and $\alpha > 0$, one obtains:

$$NR_{db}(f = f_o) < 0 \tag{105a}$$

which is found experimentally also as a negative value. From Eqn. (104a) and Eqn. (104b), one finds in the neighborhood of $f \sim f_o$:

$$NR_{db}(f > f_o) > NR_{db}(f = f_o) \tag{105b}$$

$$NR_{db}(f < f_o) > NR_{db}(f = f_o) \tag{105c}$$

and the curve $NR_{db}(\log f)$ has a local minimum at $f = f_o$. Thus, the value of f_o, the fundamental resonance frequency of the plate, may be found from the experimental curve $NR_{db}(\log f)$.

Since the fundamental resonance frequency of the plate f_o can be found from the experimental curves $NR_{db}(\log f)$, as well as the value of $NR_{db}(f = f_o)$, one is able to calculate the damping factor α of the vibrating plate from Eqn. (104b) as follows:

$$10^{\left[\frac{NR_{db}(f = f_o)}{20}\right]} = (1 - \frac{4\pi}{\mu c} f_o \alpha) \tag{106a}$$

and since $NR_{db}(f = f_o) < 0$ and $\mu = \frac{2\rho}{\rho_p h}$, one has from Eqn. (106a):

$$\alpha = \frac{\mu c}{4\pi f_o} [1 - \frac{1}{10^{\frac{-NR_{db}(f=f_o)}{20}}}] = \frac{\rho c}{2\pi \rho_p h f_o} [1 - \frac{1}{10^{\frac{-NR_{db}(f=f_o)}{20}}}] \tag{106b}$$

where $\alpha > 0$ if $NR_{db}(f = f_o) < 0$.

Thus, the values of the fundamental resonance frequency of the plate f_o and the damping factor of the vibrating plate α may be found from the experimental curves $NR_{db}(\log f)$.

C. <u>The stiffness controlled region</u>: This is the region where the frequency of the acoustic wave f is smaller than the fundamental resonance frequency of the plate f_o such that $f < f_o$. Since $|\alpha| << 1$, the first term in Eqn. (102b) could be neglected and in this region Eqn. (102b) becomes:

$$NR_{db} = 10 \log \left[\left(\frac{4\pi}{\mu c}\right)^2 \left(\frac{f_o^{\,2} - f^2}{f}\right)^2\right] =$$

$$= 20 \log \left[\frac{4\pi}{\mu c} \frac{|f_o^{\,2} - f^2|}{f}\right] =$$

$$= 20 \log \left[\frac{2\pi\rho_p h}{\rho c} \frac{|f_o^{\,2} - f^2|}{f}\right] =$$

$$= 20 \log \left[\frac{\rho_p h |f_o^{\,2} - f^2|}{f}\right] - 20 \log \left(\frac{\rho c}{2\pi}\right) =$$

$$= 20 \log \frac{|f_o^{\,2} - f^2|}{f} - 20 \log \left(\frac{\rho c}{2\pi\rho_p h}\right) \qquad (107)$$

By taking smaller acoustic wave frequencies f further away from the fundamental resonance frequency of the plate f_o, such that $f^2 << f_o^{\,2}$, Eqn. (107) becomes:

$$NR_{db} = 10 \log\left(\frac{4\pi}{\mu c}\right)^2 \left(\frac{f_o^2}{f}\right)^2 = 20 \log\left(\frac{4\pi}{\mu c} \frac{f_o^2}{f}\right) =$$

$$= 20 \log \left(\frac{4\pi f_o^2}{\mu c}\right) - 20 \log f =$$

$$= 20 \log\left[\frac{2\pi\rho_p h f_o^2}{\rho c}\right] - 20 \log f =$$

$$= C_1 - 20 \log f \tag{108}$$

In this region raising the frequency f of the acoustic wave by one octave (= doubling the frequency f) will reduce NR_{db} by 6 decibels. Drawing $NR_{db}(\log f)$ on a log-log paper will give a straight line with a slope of -20 (decrease of NR_{db} by 6 db for every octave of f) and a constant C_1 such that:

$$NR_{db}(f=1) = C_1 = 20 \log\left(\frac{2\pi\rho_p h f_o^2}{\rho c}\right) =$$

$$= 20 \log \left(\frac{2\pi\rho_p h}{\rho c}\right) + 40 \log f_o$$

$$= C + 40 \log f_o$$

$$NR_{db}(f=100) = C_1 - 40 \text{ db}$$

Once the fundamental resonance freqeuncy of the plate f_o has been found and the damping factor of the vibrating plate α has been calculated, the Noise Reduction coefficient NR_{db} can be drawn as a function of the frequency f of the acoustic wave by using Eqn. (102b) for the region of

frequencies where $|kd_1| << 1$. By drawing the curve $NR_{db}(\log f)$ on a log-log paper, one will obtain straight lines in the mass controlled region and in the stiffness controlled region.

CHAPTER VIII

FREE VIBRATIONS OF THE FINITE PANEL

In the present chapter the characteristic resonance frequencies of the vibrations of the free clamped finite panel (plate) coupled with the acoustic fluid (air) in the rigid duct will be discussed.

Let the clamped rectangular plate of dimensions "a" and "b" be situated in the rigid duct at $z = 0$ in the x-y plane. The lateral dispalcement $\eta(x,y)$ of the plate will be subject to the boundary conditions given in Eqn. (59) as follows:

$$\eta = 0 \text{ at } x = 0,\ x = a,\ y = 0,\ y = b \tag{109a}$$

$$\frac{\partial\eta}{\partial x} = 0 \text{ at } x = 0,\ x = a \text{ and } \frac{\partial\eta}{\partial y} = 0 \text{ at } y = 0,\ y = b \tag{109b}$$

Assuming harmonic time variation $e^{-i\omega t}$, the partial differential equation which governs the lateral displacement of the vibrating panel(plate) is given in Eqns. (70) as follows:

$$\nabla^4\eta - \gamma^4\eta = \frac{1}{D}\,p_z \tag{110}$$

where: $\gamma^4 = \dfrac{\rho_p h\omega^2}{D} = \dfrac{12(1-\nu^2)\rho_p\omega^2}{Eh^2}$ $\quad (1/m^4)$

p_z is the external net force per unit area in the positive z-direction (N/m^2) and the other quantities are defined in Eqn. (70a).

Two acoustic waves of mode (m,n) can propagate in the rigid duct of dimensions "a" and "b" away from the freely vibrating plate. In

accordance with Eqn. (49a), the acoustic wave of mode (m,n) propagating in the positive z-direction away from the vibrating plate, will be of the form:

$$p_+ = P_o \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{i(qz-\omega t)} \qquad z \geq 0 \quad (111a)$$

where from Eqn. (38b) one has:

$$u_{z+} = \frac{q}{\omega\rho} p_+ = \frac{qP_o}{\omega\rho} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{i(qz-\omega t)} \quad z \geq 0 \quad (111b)$$

Similarly, the acoustic wave of mode (m,n) propagating in the negative z-direction away from the vibrating plate will be of the form:

$$p_- = -P_o \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-i(qz+\omega t)} \qquad z \leq 0 \quad (112a)$$

$$u_{z-} = \frac{-q}{\omega\rho} p_- = \frac{qP_o}{\omega\rho} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} {-i(qz+\omega t)} \quad z \leq 0 \quad (112b)$$

The value of the veolcity of the plate in the lateral direction should be equal to the velocity vector of the acoustic wave on either side of the plate in accordance with Eqns. (77a) and (78a):

$$u_{zp} = \frac{d\eta}{dt} = -i\omega\eta = u_{z+}(z = 0_+) = u_{z-} \ (z = 0_-) \qquad (113)$$

Because of Eqn. (113), one has the same amplitude P_o in both waves in Eqn. (111) and in Eqn. (112). Substituting Eqn. (111b) and Eqn. (112b) in Eqn. (113), one finds the plate lateral displacement $\eta(x,y)$ in the form:

$$\eta = A \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\omega t} \qquad z = 0 \qquad (114)$$

where $A = \dfrac{iqP_o}{\omega^2 \rho}$ (m)

The value of the lateral displacement $\eta(x,y)$ found in Eqn. (114) is directly related to the acoustic wave mode (m,n) propagating in the rigid duct. This value of $\eta(x,y)$ in Eqn. (114) obeys the boundary conditions, Eqn. (109b), for a clamped plate at the edges. However, the value of $\eta(x,y)$ at Eqn. (114) does not obey the boundary conditions, Eqn. (109a), for a clamped plate at the edges. This requirement that $\eta = 0$ at the edges of the clamped plate will introduce an interaction between the different modes (m,n) given in Eqns. (111), (112) and (114) and the fundamental mode (m = 0, n = 0) of the acoustic wave propagating in the rigid duct and discussed previously. The value of $A_{m,n}$ in Eqn. (114) for each mode (m,n) will be related to all the other modes (m,n) and to $A_{0,0}$ of the fundamental mode. However, in the present report we are interested in finding the values of the characteristic frequencies of vibrations and not their amplitudes. We are able to use Eqn. (114) for finding these characteristic resonance frequencies of vibration, but not their amplitudes.

From Eqn. (111a) and Eqn. (112a), one obtains at z = 0:

$$p_z = (p_- - p_+)_{z=0} = -2P_o \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\omega t} \qquad (115)$$

Substituting Eqns. (114) and (115) in Eqn. (110), one obtains after cancelling the common functions:

$$[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2]^2 - \gamma^4 = \frac{1}{D} \frac{-2P_o}{A} = \frac{1}{D} \frac{i2\omega^2 \rho}{q} \qquad (116a)$$

Substituting $\mu = \frac{2\rho}{\rho_p h}$ and $\gamma^4 = \frac{\rho_p h\omega^2}{D}$, one obtains from Eqn. (116a):

$$[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2]^2 - \gamma^4 = i\,\frac{\mu\gamma^4}{q} \qquad (116b)$$

For an acoustic wave of mode (m,n) in a rigid duct, one has from Eqn. (50b):

$$q = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} = \sqrt{(\frac{\omega}{c})^2 - (\frac{m\pi}{a})^2 - (\frac{n\pi}{b})^2} \qquad (116c)$$

Substituting Eqn. (116c) in Eqn. (116b), one has:

$$[k_c^4 - \gamma^4]\sqrt{k^2 - k_c^2} = i\mu\gamma^4 \qquad (117a)$$

where $k_c^2 = (\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2$ and k_c is the cut off wave number for the acoustic wave mode (m,n) in the rigid duct given in detail in Eqns. (40) and (50). Eqn. (117a) may be rewritten alternatively in the form:

$$[k_c^4 - \gamma^4]\sqrt{k_c^2 - k^2} = \mu\gamma^4 \qquad (117b)$$

where $\mu = \frac{2\rho}{\rho_p h}$ is the coupling coefficient between the acoustic waves and the plate vibrations in the duct.

Taking $k = \frac{\omega}{c}$ and $\gamma^4 = \frac{\rho_p h}{D}\,\omega^2$ and substituting in Eqn. (117b), one obtains:

$$[k_c^4 - \frac{\rho_p h}{D}\,\omega^2]\sqrt{k_c^2 - (\frac{\omega}{c})^2} = \mu\,\frac{\rho_p h}{D}\,\omega^2 \qquad (118a)$$

By squaring both sides of Eqn. (118a), one obtains:

$$[k_c^4 - \frac{\rho_p h}{D}\omega^2]^2 [k_c^2 - \frac{\omega^2}{c^2}] = \mu^2 (\frac{\rho_p h}{D})^2 \omega^4 \tag{118b}$$

Eqn. (118b) is a cubic algebraic equation in ω^2 for the characteristic resonance frequencies for mode (m,n). It has three distinct positive roots for the resonance frequencies ω of the coupled system of the clamped plate and the acoustic waves in the rigid duct. It can be solved for numerical values by using the standard cubic equation solution for numerical approximation.

An alternative algebraic solution of Eqn. (118b) for the case considered in the present report will be based on the following inequalities:

$$c^2 \gg k_c^2 = [(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] \tag{119a}$$

$$k_c^2 \gg \mu^2 = (\frac{2\rho}{\rho_p h})^2 \tag{119b}$$

where in Eqn. (119b) the fundamental mode (m=0, n=0) is excluded. Opening the brackets on the left hand side, one of the terms will be much larger than the term on the right hand side because of Eqn. (119b) where $k_c^2 \gg \mu^2$:

$$\ldots + (\frac{\rho_p h}{D})^2 k_c^2 \omega^4 + \ldots \gg \mu^2 (\frac{\rho_p h}{D})^2 \omega^4$$

As a result, for the first approximation, the coupling coefficient term on the right hand side of Eqn. (118b) could be neglected, and Eqn. (118b) becomes for the first approximation:

$$[k_c^{\,4} - \frac{\rho_p h}{D}\,\omega^2]^2[k_c^{\,2} - \frac{\omega^2}{c^2}] = 0 \tag{120}$$

For the first approximation in Eqn. (120) the acoustic resonance frequency and the plate vibrations resonance frequencies are uncoupled. The acoustic resonance freqeuncy ω_1 can be obtained from Eqn. (120) as follows:

$$k_c^{\,2} - \frac{\omega_1^{\,2}}{c^2} = 0; \quad \omega_1 = ck_c = c\sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2} \tag{121a}$$

where the acoustic resonance frequency ω_1 for the first approximation Eqn. (121a) is equal to the cut-off frequency $\omega_c = ck_c$ of the acoustic wave of the same mode (m,n) in a rigid duct given in Eqn. (50). The corresponding plate vibrations resonance frequencies ω_2 and ω_3 can be obtained from Eqn. (120) as follows:

$$[k_c^{\,4} - \frac{\rho_p h}{D}\,\omega^2]^2 = 0; \; \omega_2 = \omega_3 = \sqrt{\frac{D}{\rho_p h}}\,k_c^{\,2} = \sqrt{\frac{D}{\rho_p h}}[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2] \tag{121b}$$

where the plate vibrations resonance frequencies ω_2 and ω_3 for the first approximation, Eqn. (121b), appear as a double root of the equation, and is equal to the resonance frequency of the mode (m,n) of the plate vibrations given in Eqn. (66). For the first approximation, when the coupling effect between the plate and the fluid (air) is neglected, one finds the acoustic resonance frequency ω_1 in Eqn. (121a) as a single root of the Eqn. (120), and the plate resonance frequencies $\omega_2 = \omega_3$ in Eqn. (121b) as a double root of the Eqn. (120).

For the second approximation of ω_1, the small coupling term on the right hand side of Eqn. (118b) will be included. Substituting the value of ω_1 found in Eqn. (121a) in the first and the third terms of Eqn. (118b), one will obtain the following equation for the second approximation of ω_1:

$$[k_c^4 - \frac{\rho_p h}{D}(ck_c)^2]^2 [k_c^2 - \frac{\omega_1^2}{c^2}] = \mu^2 (\frac{\rho_p h}{D})^2 (ck_c)^4$$

which may be rewritten in the form:

$$k_c^2 - \frac{\omega_1^2}{c^2} = \frac{\mu^2 (\frac{\rho_p hc^2}{D})^2}{[k_c^2 - \frac{\rho_p hc^2}{D}]^2} = \frac{\mu^2}{[1 - \frac{Dk_c^2}{\rho_p hc^2}]^2}$$

Using Eqn. (119a) where $k_c^2 << c^2$, one could expand the previous denominator to give:

$$k_c^2 - \frac{\omega_1^2}{c^2} = \mu^2 [1 + \frac{2Dk_c^2}{\rho_p hc^2}]$$

Rearranging, one obtains:

$$\omega_1^2 = c^2 k_c^2 - c^2 \mu^2 (1 + \frac{2Dk_c^2}{\rho_p hc^2}) = c^2 k_c^2 [1 - \frac{\mu^2}{k_c^2}(1 + \frac{2Dk_c^2}{\rho_p hc^2})]$$

Using Eqn. (119b), where $\mu^2 << k_c^2$, one could expand the previous equation to give:

$$\omega_1 = ck_c \left[1 - \frac{1}{2}\frac{\mu^2}{k_c^2} - \frac{D\mu^2}{\rho_p hc^2}\right] \tag{122a}$$

Since from Eqn. (119) $\mu^2 << k_c^2 << c^2$, one could neglect the last term, as small of the second order of magnitude, for the present approximation, and obtain:

$$\omega_1 = ck_c \left[1 - \frac{1}{2}\frac{\mu^2}{k_c^2}\right] = ck_c - \frac{1}{2}\frac{c}{k_c}\mu^2 \tag{122b}$$

Equation (122b) gives the second approximation for the acoustic resonance frequency ω_1, where the coupling coefficient between the plate and the fluid (air) has been included. From Eqn. (122b), one sees that the resonance frequency ω_1 of the acoustic wave is smaller than the cut-off frequency $\omega_c = ck_c$ of the same mode (m,n) of the acoustic wave propagation in the rigid duct:

$$\omega_1 < ck_c = c\,\frac{\omega_c}{c} = \omega_c \qquad\qquad f_1^{m,n} < f_c^{m,n} \tag{122c}$$

Therefore, the acoustic resonance ω_1 of the plate-air resonance system will establish acoustic wave mode (m,n) of frequency ω_1 in the rigid duct, which will not propagate, but will attenuate exponentially as one goes further and further away from the free vibrating plate and its coupling with the fluid (air).

For the second approximation of ω_2 and ω_3, the small coupling term on the right hand side of Eqn. (118b) will be included. Substituting the value of $\omega_2 = \omega_3$, found in Eqn. (121b), in the second and third terms of Eqn. (118b), one will obtain the following equation for the second approximation of ω_2 and ω_3:

$$[k_c^4 - \frac{\rho_p h}{D}\omega_{2,3}^2]^2[k_c^2 - \frac{Dk_c^4}{\rho_p hc^2}] = \mu^2(\frac{\rho_p h}{D})^2(\frac{Dk_c^4}{\rho_p h})^2$$

which may be rewritten in the form:

$$[k_c^4 - \frac{\rho_p h}{D}\omega_{2,3}^2]^2 = \frac{\mu^2 k_c^8}{[k_c^2 - \frac{Dk_c^4}{\rho_p hc^2}]} = \frac{\mu^2 k_c^6}{[1 - \frac{Dk_c^2}{\rho_p hc^2}]}$$

Using Eqn. (119a) where $k_c^2 << c^2$, one could expand the previous denominator to give:

$$[k_c^4 - \frac{\rho_p h}{D}\omega_{2,3}^2]^2 = \mu^2 k_c^6[1 + \frac{Dk_c^2}{\rho_p hc^2}]$$

Taking the square root of both sides and expanding, since $k_c^2 << c^2$, one has:

$$k_c^4 - \frac{\rho_p h}{D}\omega_{2,3}^2 = \mp\mu\, k_c^3[1 + \frac{Dk_c^2}{2\rho_p hc^2}]$$

Rearranging, one obtains:

$$\frac{\rho_p h}{D}\omega_{2,3}^2 = k_c^4 \pm \mu k_c^3(1 + \frac{Dk_c^2}{2\rho_p hc^2})$$

$$= k_c^4[1 \pm \frac{\mu}{k_c}(1 + \frac{Dk_c^2}{2\rho_p hc^2})]$$

which will give:

$$\omega_{2,3}^2 = \frac{D}{\rho_p h}k_c^4[1 \pm \frac{\mu}{k_c}(1 + \frac{Dk_c^2}{2\rho_p hc^2})]$$

Using the inequality in Eqn. (119b), $\mu^2 << k_c^2$, one could expand the previous equation to give:

$$\omega_{2,3} = \sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2} \left[1 \pm \frac{1}{2}\frac{\mu}{k_c}\left(1 + \frac{D k_c^{\,2}}{2\rho_p h c^2}\right)\right]$$

Taking the positive sign for ω_2 and the negative sign for ω_3, one has:

$$\omega_2 = \sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2}\left[1 + \frac{1}{2}\frac{\mu}{k_c} + \frac{D k_c \mu}{4\rho_p h c^2}\right] \tag{123a}$$

$$\omega_3 = \sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2}\left[1 - \frac{1}{2}\frac{\mu}{k_c} - \frac{D k_c \mu}{4\rho_p h c^2}\right] \tag{123b}$$

Using Eqn. (119a), where $k_c^{\,2} << c^2$, one could neglect the last term as small of the second order of magnitude for the present approximation, and obtain:

$$\omega_2 = \sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2}\left[1 + \frac{1}{2}\frac{\mu}{k_c}\right] = \sqrt{\frac{D}{\rho_p h}}\left[k_c^{\,2} + \frac{1}{2}\mu k_c\right] \tag{124a}$$

$$\omega_3 = \sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2}\left[1 - \frac{1}{2}\frac{\mu}{k_c}\right] = \sqrt{\frac{D}{\rho_p h}}\left[k_c^{\,2} - \frac{1}{2}\mu k_c\right] \tag{124b}$$

Equations (124) give the second approximation for the plate resonance frequencies ω_2 and ω_3, where the coupling coefficient between the plate and the fluid (air) has been included. In this case, one obtains two single roots, $\omega_2 \neq \omega_3$, of Eqn. (118b), which become a double root, $\omega_2 = \omega_3$, only when the coupling coefficient has been neglected. The difference in the frequency between the two roots may be found from Eqns. (124) to give:

$$\omega_2 - \omega_3 = \sqrt{\frac{D}{\rho_p h}}\, \mu k_c \tag{124c}$$

The three discrete resonance frequencies ω_1, ω_2, ω_3 for the mode (m,n) of the coupled system of the clamped plate and the fluid (air) in the rigid duct have been given in Eqns. (122b), (124a) and (124b) under the assumption, Eqn. (119), that $\mu^2 << k_c^{\ 2} << c^2$. One could rewrite Eqn. (118b) therefore in the form:

$$(\omega^2 - \omega_1^{\ 2})\ (\omega^2 - \omega_2^{\ 2})\ (\omega^2 - \omega_3^{\ 2}) = 0 \tag{125}$$

Since $\omega_1^{\ 2}$, $\omega_2^{\ 2}$, $\omega_3^{\ 2}$ are the roots of Eqn. (118b). Substituting ω_1, ω_2, ω_3 from Eqns. (122b), (124a), (124b) respectively in Eqn. (125) one obtains Eqn. (118b), if the terms $\frac{\mu^2}{k_c^{\ 2}} << 1$ are neglected, in accordance with Eqn. (119b) and our present approximation. This checks the results found for ω_1, ω_2, ω_3 under the present approximation. Comparing Eqn. (122b) with Eqn. (124), if one had $k_c <<\ c$ for the present cases under consideration, one will obtain:

$$\omega_2 << \omega_1 < \omega_c \qquad\qquad \omega_3 << \omega_1 < \omega_c \tag{126}$$

Thus, the two plate discrete resonance frequencies are below the cut-off frequency of the corresponding acoustic wave mode (m,n) in the rigid duct and will attenuate exponentially. However, since the plate resonance amplitude $A_{m,n}$ is coupled with $A_{0,0}$ through the boundary conditions $\eta = 0$ at the edges, as discussed previously, the plate resonance at frequencies ω_2 and ω_3 will produce also the fundamental acoustic mode in the duct because of the above interaction, and the fundamental

acoustic mode in the duct will propagate at any frequency, regardless of the value of the cut-off frequency.

Let the fundamental mode of the acoustic waves in a rigid duct be incident on a panel (plate) situated at z = 0. If one takes the solution of the plane wave incident on an infinite plate given in Eqns. (81), one sees that the amplitude of the plate vibration is given in Eqns. (74a) and (81a), and is a function of time only and not of position on the plate. By introducing the duct rigid wall, parallel to the lateral direction of the plate, one introduces the boundary conditions, Eqn. (109), to the clamped plate, but does not effect the acoustic plane wave, which is identical with the fundamental mode. In accordance with Eqn. (109a), the clamped plate is not allowed to move at the edges; but according to the solution for infinite plate in Eqn. (81a), the plate does move at the edge. Since at the edges of the clamped plate $\eta = 0$, and the clamped plate cannot move at the clamped edges, the plate starts to vibrate at higher modes (m,n) in such a way that the total net movement of the edges obey $\eta = 0$. Thus, it is necessary that the clamped plate will vibrate at higher modes in order to compensate for the movement of the edges because of the plane acoustic wave, and the vibration is such the $\eta = 0$ is kept at the edges of the clamped plate. Thus, the existence of higher order acoustic modes (m,n) in the duct are necessary for the clamped plate case, in order to match the boundary conditions $\eta = 0$ at the edges of the plate; the higher order modes exist even if the incident aocustic wave on the clamped plate is a perfect fundamental acoustic wave, a completely uniform plane wave within the duct rigid walls, coming from a perfect source.

CHAPTER IX

THE BERANEK TUBE RECEIVING CHAMBER

The receiving chamber of the Beranek tube used in the experimental set up includes highly nonuniform lossy material of different kinds. The total length of the receiving chamber is about 90" inches with inside cross-section of 18" x 18". The inside tube contains 18" inches of loosely packed foam behind the receiving microphone, with additional length of 9" of loosely packed foam in wedge form on both sides of the ..eiving microphone. The loose foam on both sides of the receiving microphone and behind it absorbs high frequencies in the neighborhood of the receiving microphone, and reduces transverse standing waves.

Bel..nd the foam there are four highly absorbent fiberglass wedges with overall length of 60" inches, which include a rectangular base 24" inches long. The wedges are 9 sq. inch and made from semi-rigid fiberglass to help preserve their shape. The density of the fiberglass used in the wedges is 3 lb/ft^3. The wedges are positioned in the tube with their blades alternated to present a maximum absorbent area to the sound. The wedges are followed by 8" inches of cotton and 4" inches of fiberglass to the back panel. The cotton and fiberglass packed between the fiberglass wedges and the back wooden panel help absorb the lower frequencies.

In order to simplify matters from theoretical point of view, it will be assumed that the absorption material is uniform and presents an average uniform absorption effect to the incident and reflected acoustic waves. It will be further assumed that only plane waves propagate in the

absorption material. Since the higher order modes attenuate much faster than the plane waves in the duct, because of losses at the walls of the duct, this assumption should not change much the basic physical phenomena. Once the basic absorption phenomena of the incident and reflected waves has been analyzed and compared with experiment, additional corrections can be made by taking into account the nonuniformity of the absorption material and the higher order modes of propagation.

Let a uniform plane acoustic wave be incident in the positive z-direction in a lossy medium:

$$p_i = p_I e^{-\alpha z_d} e^{i(kz-\omega t)} = p_I e^{(ik-\alpha)z-i\omega t} \tag{127a}$$

where $k = \omega/c$ and α represents the attenuation factor in the lossy medium. Substituting Eqn. (127a) in Eqn. (20b), one obtains:

$$u_{zi} = p_I \frac{ik-\alpha}{i\omega\rho} e^{-\alpha z} e^{i(kz-\omega t)} = p_I \frac{1+i\alpha/k}{\rho c} e^{+(ik-\alpha)z-i\omega t} \tag{127b}$$

Let a uniform plane acoustic wave be reflected in the negative z-direction in a lossy medium:

$$p_r = p_R e^{\alpha z} e^{-i(kz+\omega t)} = p_R e^{-(ik-\alpha)z-i\omega t} \tag{128a}$$

Substituting Eqn. (128a) in Eqn. (20b), one obtains:

$$u_{zr} = -p_R \frac{(ik-\alpha)}{i\omega\rho} e^{\alpha z} e^{-i(kz+\omega t)} = -p_R \frac{1+i\alpha/k}{\rho c} e^{-(ik-\alpha)z-i\omega t} \tag{128b}$$

Taking the back wooden panel to be situated at $z = L$, one has there $u_n = 0$ and from Eqns. (127b) and (128b) one obtains, after cancelling the common factors:

$$u_n = 0 = u_{zi}\Big|_{z=L} + u_{zr}\Big|_{z=L} = p_I e^{(ik-\alpha)L} - p_R e^{-(ik-\alpha)L} \qquad (129a)$$

From Eqn. (129a) one has

$$p_R = p_I \; e^{2(ik-\alpha)L} \qquad (129b)$$

which is identical with Eqn. (28b) for $\alpha = 0$. Assuming in the present case that the attenuation factor is small, $\alpha/k << 1$, one is able to continue the analysis similarly to Eqns. (29) to (35). The effect of the attenuation factor $\alpha \neq 0$ is to reduce the value of the resonance frequencies calculated for $\alpha = 0$.

Rewriting Eqn. (35), one has:

$$p_{max} \; (z = z_0) = 2\sqrt{2} \; p_I \left|\cos[k(L-z_0) - \phi(\omega)]\right| \qquad (130)$$

For the case $\phi(\omega) = 0$, one requires for $p_{max} = 0$ at $z = z_o$ that:

$$k(L - z_0) = \frac{\omega}{c} (L - z_0) = \frac{2\pi f}{c} (L - z_0) = \frac{2\pi}{\lambda} (L - z_0) = (2n + 1)\frac{\pi}{2} \qquad (131a)$$

where $n = 1,2,3 \ldots$ From Eqn. (131a), one obtains the characteristic frequencies for minimum value of the pressure p at the receiver microphone situated at $z = z_0$:

$$f_n = \frac{c}{L-z_0}\,\frac{2n+1}{4} \qquad \text{for} \qquad \phi(\omega) = 0 \tag{131b}$$

For the case $\phi(\omega) = \frac{-\pi}{2}$, one requires in Eqn. (130a) for $p_{max} = 0$ at $z = z_0$ that:

$$k(L - z_0) + \frac{\pi}{2} = \frac{2\pi f}{c}(L - z_o) + \frac{\pi}{2} = (2n + 1)\,\frac{\pi}{2} \tag{132a}$$

where n = 1,2,3 . . . From Eqn.(132a), one obtains similarly:

$$f_n = \frac{c}{L-z_0}\,\frac{n}{2} \qquad \text{for} \qquad \phi(\omega) = \frac{-\pi}{2} \tag{132b}$$

In our experimental set up the distance of the wooden back panel from the test aluminum panel is L = 107.5" and the distance of the receiver microphone is z_0 = 8". Therefore, one has in our experiment:

$$L - z_o = 107.5'' - 8.0'' = 99.5'' = 2.527 \text{ m} \tag{133a}$$

Taking the acoustic velocity c for T = 21°C ≅ 70°F from Eqn. (6) to be:

$$c = 343.8 \text{ m/sec} \tag{133b}$$

Using Eqn. (133) in Eqns. (131b) and (132b), one obtains:

$$f_n = 68.0(n + \tfrac{1}{2}) \text{ (1/sec)} \qquad \text{for} \qquad \phi(\omega) = 0 \tag{133c}$$

$$f_n = 68.0\, n \text{ (1/sec)} \qquad \text{for} \qquad \phi(\omega) = \frac{-\pi}{2} \tag{133d}$$

for n = 1,2,3 . . .

From the experimental results it is found that Eqn. (133d) represents the results much better than Eqn. (133c). Thus, a phase shift of $\phi(\omega) = -90°$ is introduced between the incident plane wave and the reflected plane wave by the wooden back panel in accordance with Eqn. (33). As indicated earlier, the attenuation factor $\alpha \neq 0$ in the lossy medium will introduce a shift of the actual resonance frequencies found experimentally to lower frequencies than calculated in accordance with Eqn. (133d), which is a well established phenomena.

Let us now calculate the resonance frequency of the first even vibration mode (m = 2, n = 2) of the wooden back panel in accordance with Eqn. (55c) and Eqn. (66d), which are reproduced here for reference:

$$f^{wood}_{m,n} = \frac{\pi}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sqrt{\frac{D}{\rho_p h}}; \quad D = \frac{Eh^3}{12(1-\nu^2)} \tag{134a}$$

Taking for our experimental set up requirements a = b and m = n, one obtains from Eqn. (134a):

$$f^{wood}_{m,m} = \pi\left(\frac{m}{a}\right)^2 h\sqrt{\frac{E}{12\rho_p(1-\nu^2)}} \tag{134b}$$

In the present experimental set up the wooden back panel has the following parameters:

$$a = 18'' = 0.4572\text{m}; \quad h = 0.75'' = 0.019\text{m} \tag{135a}$$

$$\rho_p = \frac{6.89\ \text{lb}}{243\ \text{inch}^3} = 49.0\ \text{lb/ft}^3 = 785.0\ \text{Kg/m}^3 \tag{135b}$$

Since the Young's modulus of elasticity E is not available for the wooden back panel, it will be assumed to be identical with poplar wood as follows:

$$E = 1.0 \times 10^{10} \quad (N/m^2) \qquad 1 - \nu^2 \cong 0.9 \tag{135c}$$

Substituting Eqns. (135) in Eqn. (134b), one obtains the resonance frequency of the first even vibrational mode $m = 2$ of the wooden back panel:

$$f_{2,2}^{wood} = 1{,}240 \text{ 1/sec} \tag{135d}$$

The experimental curve in the present paper shows a very sharp resonance frequency at $f = 1{,}260$ Hz which could be identified with the above vibrational mode $m = 2$ in Eqn. (135d) of the wooden back panel. The above resonance represents interaction between the incident acoustic and reflected acoustic waves with the vibrational modes of the aluminum panel in front, the wooden back panel and the receiver microphone. A more detailed analysis of the wooden back panel will be given in the next chapter.

When two resonance systems interact with each other the resonance frequency ω of the combined system could be given similarly to Eqn. (125):

$$(\omega^2 - \omega_1{}^2)\ (\omega^2 - \omega_2{}^2) = \mu_0{}^4 \tag{136a}$$

where ω_1 is the resonance frequency of the first system, ω_2 is the resonance frequency of the second system ($\omega_2 > \omega_1$) and μ_o represents the

C-2

coupling factor between the systems. When there is no coupling $\mu_0 = 0$, the resonance frequencies are:

$$\mu_0 = 0; \quad \omega = \omega_- = \omega_1 \text{ and } \omega = \omega_+ = \omega_2 > \omega_1 \tag{136b}$$

In case of coupling between the systems $\mu_o \neq 0$, Eqn. (136a) can be rewritten in the following form:

$$\omega^4 - (\omega_2^2 + \omega_1^2)\omega^2 + (\omega_2^2\omega_1^2 - \mu_0^4) = 0 \tag{137a}$$

Solving the quadratic Eqn. (137a) for ω^2 for $\mu_o \neq 0$, one has:

$$\omega_+^2 = \frac{1}{2}\left[(\omega_2^2 + \omega_1^2) + \sqrt{(\omega_2^2 - \omega_1^2)^2 + 4\mu_0^4}\right] > \omega_2^2 \tag{137b}$$

$$\omega_-^2 = \frac{1}{2}\left[(\omega_2^2 + \omega_1^2) - \sqrt{(\omega_2^2 - \omega_1^2)^2 + 4\mu_0^4}\right] < \omega_1^2 \tag{137c}$$

where Eqns. (137b) and (137c) represent the natural resonance frequencies of the coupled systems, when the oscillators are no longer isolated. Thus, the coupling between the systems causes the higher resonance frequency ω_+ to shift higher, the lower resonance frequency ω_- to shift lower. When there are several resonance systems interacting with each other, it is not surprising sometimes to find that the experimental resonance frequencies are shifted as compared to the theoretical resonance frequencies calculated for each system by itself, without taking into account the coupling factor $\mu_0 \neq 0$.

CHAPTER X

EXPERIMENTAL RESULTS

In the present chapter we will apply the theory which has been given in this report to analyze some of the outstanding apsects of the experimental results given in Figure 1. The aluminum panel which gives the noise reduction in Figure 1 has the following dimensions and properties:

Material: Alclad 2024T3 Aluminum

Thickness: $h = 0.025'' = 0.635$ mm

Width x Axis: $a = 18'' = 0.4572$ m

Width y Axis: $b = 18'' = 0.4572$ m

Some mechanical properties of aluminum are as follows:

Density $= \rho_p = 2.7 \times 10^3$ Kg/m^3

Young modulus of elasticity $= E = 7.0 \times 10^{10}$ N/m^2

Poisson's ratio $= \nu = 0.3$

$$1 - \nu^2 = 0.91$$

The experiments were done in Lawrence, Kansas at about 1,000 ft above sea level under the following conditions of the air:

Temperature $= T = 21°C \cong 70°F = 294.15°K$

Pressure $= P = 0.97735$ atmosphere $= 97{,}735$ N/m^2

Density $= \rho = 1.1575$ Kg/m^3

Velocity of Acoustic Waves $= c = 343.8$ m/sec

The analysis of the experimental results in Figure 1 will be divided to several parts according to the particular aspect of the physical phenomena to be discussed.

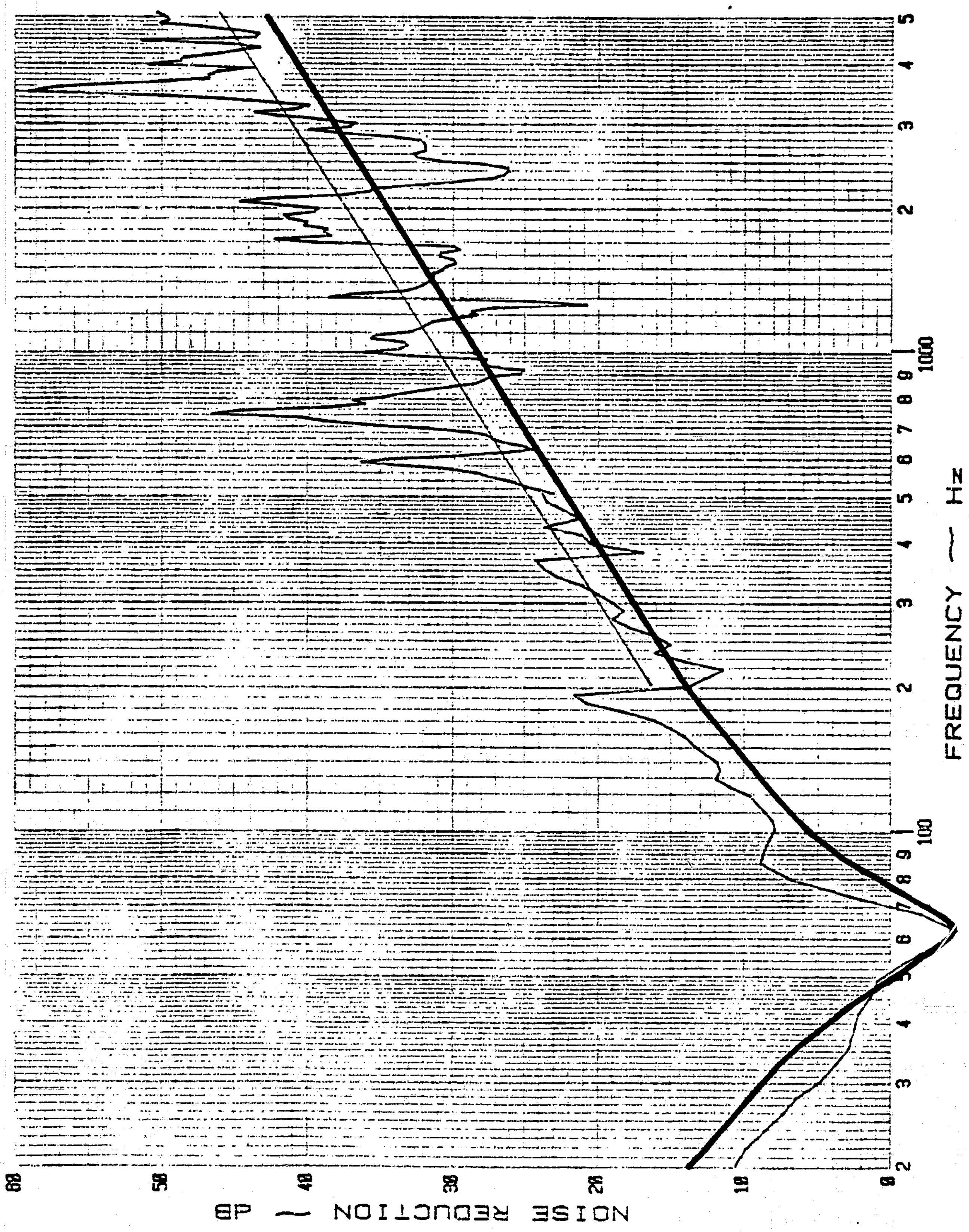

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FIGURE 1

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A. Plane Wave on the Finite Panel

Since for the transmitter microphone $d_1 = 1$ cm one has $|kd_1| << 1$, the general characteristics of the Noise Reduction curve of Figure 1 could be found by using Eqn. (102b), which is rewritten here:

$$NR_{db} = 10 \log[(1 - \frac{4\pi}{\mu c} \frac{f_0^2 \alpha}{f})^2 + (\frac{4\pi}{\mu c})^2 (\frac{f^2 - f_0^2}{f})^2] \qquad (138)$$

The coupling parameter μ between the aluminum panel and the air is:

$$\mu = \frac{2\rho}{\rho_p h} = \frac{2 \times 1.1575}{2.7 \times 0.635} = 1.350 \qquad (1/m) \qquad (139)$$

From Figure 1 one sees that the fundamental resonance frequency of the plate is:

$$f_0 = 63 \text{ Hz} \qquad (140a)$$

at which point the Noise Reduction is:

$$NR_{db}(f = f_0) = -4.5 \text{ db} \qquad (140b)$$

Using the experimental values of Eqn. (140) in Eqn. (106b), one is able to calculate the damping factor α by using Eqn. (139) and $c = 343.8$ m/sec:

$$\alpha = \frac{\mu c}{4\pi f_0}\left[1 - \frac{1}{10^{\frac{-NR_{db}(f=f_0)}{20}}}\right] \approx \frac{1.350 \times 343.8}{4\pi \times 63}\left[1 - \frac{1}{10^{\frac{4.5}{20}}}\right] =$$

$$= 0.58626\left[1 - \frac{1}{10^{0.225}}\right] = 0.58626\left[1 - \frac{1}{1.6788}\right] = 0.2370 \qquad (140c)$$

Substituting Eqns. (139) and (140) in Eqn. (138), one obtains:

$$NR_{db} = 10 \log\left[\left(1 - \frac{25.468}{f}\right)^2 + 7.33 \times 10^{-4}\left(\frac{f^2 - 63^2}{f}\right)^2\right] \qquad (141)$$

where NR_{db} is given as a function of the frequency for Figure 1. Table B gives the results for some of the frequencies calculated from Eqn. (141).

TABLE B

NOISE REDUCTION VALUES OF EQN. (141)

f	NR_{db}
20 Hz	+13.7 db
30 Hz	8.9 db
40 Hz	+ 4.3 db
50 Hz	- 0.6 db
63 Hz	- 4.5 db
70 Hz	- 2.7 db
80 Hz	+ 0.6 db
100 Hz	5.1 db
200 Hz	13.9 db
400 Hz	20.5 db
1000 Hz	28.7 db
5000 Hz	42.6 db

Figure 1 gives the experimental Noise Reduction curve for the aluminum panel discussed here. The thin line gives the experimental ratio of pressures in decibels between the receiver microphone located 8" behind the aluminum panel to the transmitter microphone located 1 cm in front of the aluminum panel, as a function of frequency. The thin straight line in Figure 1 represents a straight line least square error average of the experimental curve. The heavy line represents the theoretical curve of the present paper, with the values given in Table B and calculated in accordance with Eqn. (141). As shown above, the theoretical curve is in accordance with the experimental fundamental frequency of resonance f_0, and the experimental Noise Reduction value at this frequency, so that the theoretical curve is passing through this point $f = f_0$ on the experimental curve.

The difference between the experimental curve (thin wiggly line) and the theoretical curve (thick black line) represents two major effects:

1. In the higher frequency region this difference represents the effect of the higher order modes and the different resonance phenomena.
2. In the lower frequency region this difference represents the effect of the near zone fields next to the panel.

B. The Fundamental Resonance Frequency (f_0 = 63 Hz)

The fundamental experimental resonance frequency f_0 = 63 Hz in Figure 1 and in Eqn. (140a) is the result of interaction between two major resonance phenomena:

1. The standing plane acoustic wave which propagates along the length of the Beranek tube and is reflected by the wooden back panel. This basic resonance frequency is given in Eqn. (133d) $f_1^{cavity} = 68$ Hz. However, this resonance frequency is shifted downward because of its interaction with the vibration resonance frequency of the wooden back panel.
2. The first even resonance mode of the aluminum panel vibrations, which excite acoustic waves at $f_{2,2}^{plate} = 58.8$ Hz (see later). Although this acoustic wave mode does not propagate in the high pass filter duct, because of its large wavelength it has a definite effect on the receiver microphone as a near zone acoustic wave field excited by the first even mode of the aluminum panel vibrations.

As a result of the interaction between the above two basic resonance effects, one obtains the fundamental experimental resonance frequency at $f_0 = 63$ Hz.

The evidence for the above interaction can be found in other experimental data:

a. From Eqn. (132b) it is found that f_1^{cavity} is inversely proportional to the total length L of the Beranek tube since $|z_0| << L$. Experimental data has shown that by decreasing the length of the Beranek tube the fundamental resonance frequency f_0 increases. The same will happen even if there is no wooden back panel and the duct end is open to infinite space. The open duct end causes reflection of the acoustic wave as much because of the sudden change of impedance from the duct to infinite space.

b. From Eqn. (134b) one finds that the aluminum panel resonance frequency of the first even mode $f_{2,2}^{plate}$ is directly proportional to the thickness of the plate h. However, experimental data has shown that the fundamental resonance frequency f_0 does not change by much for changing thickness of the aluminum panel. This is because the resonance effect (a) of the standing plane acoustic wave along the length of the Beranek tube does not change, and it is very dominant in the interaction between the two resonance phenomena.

C. <u>The Cavity Resonance</u>

The resonance effect of the standing plane acoustic wave along the length of the Beranek tube will be called the cavity resonance effect and will be discussed in the present section. The cavity resonance effect is described by Eqn. (133d) as follows:

$$f_s^{cavity} = 68\ s \qquad (s = 1,2,3\ .\ .\ .) \tag{142}$$

In the following Table C the theoretical numerical values of Eqn. (142) as well as the corresponding resonance frequency experimental values from Figure 1 will be given. Theoretical numerical values from Eqn. (142) with no corresponding experimental values from Figure 1 will be omitted from Table C.

TABLE C

CAVITY RESONANCE FREQUENCIES

s	$f_s^{cavity} = 68\ s$	Experimental
1	68	63
2	136	129
*3	204	*194
4	272	278
6	408	400
7	476	431
*9	612	*592
10	680	660
12	816	798
14	952	950
*15	1,020	*1,000
*16	1,088	*1,080
18	1,224	1,220
*19	1,292	*1,300
22	1,496	1,470
24	1,632	1,600
*25	1,700	*1,720
26	1,768	1,790
27	1,836	1,830
29	1,972	1,940

Note: * Denotes major experimental upward spikes.

TABLE C (continued)

s	$f_s^{cavity} = 68\ s$	Experimental
*30	2,040	*2,070
32	2,176	2,200
38	2,584	2,600
43	2,924	2,900
47	3,196	3,180
*52	3,536	*3,500
55	3,740	3,760
*59	4,012	*4,000
60	4,080	4,070
65	4,420	4,400
*66	4,488	*4,480
72	4,896	4,860

Note: * Denotes major experimental upward spikes.

Some of the experimental values listed in Table C are only a sharp break in the experimental curve. Most of the experimental values indicated in Table C are upward spikes in the experimental curve given in Figure 1. The resonance frequency f_s^{cavity} in Eqns. (142) and (133d) are derived from Eqn. (130) under the condition that $p_{max}(z = z_0) = 0$, which means that the pressure at the receiver microphone is zero. Therefore, at the resonance frequencies listed in Table C, the resonance spikes in Figure 1 are upward, representing a strong noise reduction,

since the transmitted acoustic wave at the receiver microphone is taken to be zero at these frequencies. However, because of the near zone fields of the higher order modes, the pressure at the receiver microphone is not quite zero, and therefore the resonance upward spikes of the experimental curve in Figure 1 are finite.

The agreement between the theoretical cavity resonance frequencies and the experimental resonance frequencies in Table C is relatively good since no interaction effect has been taken into account in the theory, such effects usually shift the calculated resonance frequency. The interaction effects include the near zone fields of the higher order modes excited by the panel, and the interaction effects with the wooden back panel and the wooden walls of the duct, the effects of which have been neglected by assuming that the wooden walls are infinitely massive and do not vibrate. Particularly large sharp upward spikes of this phenomena appear at the f_s^{cavity} for s = 3, 9, 15, 16, 19, 25, 30, 52, 59, 66, which are denoted by a star at Table C.

D. The Acoustic Resonance

It has been shown in Chapter VIII that the free vibrations of the panel will establish acoustic wave resonance modes (m,n) of circular frequency ω_1 in the rigid duct, the value of which is given in Eqn. (122a) and Eqn. (122b). This acoustic wave resonance modes will not propagate in the duct, but will attenuate exponentially as one goes further and further away from the free vibrating plate coupled with the air system, because the resonance frequencies of these modes are slightly below the cut-off frequency of the corresponding higher order modes in the duct, as shown in Eqn. (122c). Thus, at these acoustic

resonance frequencies of the free vibrating plate, the acoustic pressure from these modes will be higher ~~at~~ the transmitter microphone than at the receiver microphone, since the transmitter microphone is much closer to the aluminum panel than the receiver microphone. As a result, at the acoustic resonance frequencies one will get upward spikes in the experimental curve representing resonance "noise reduction."

Equation (121a) gives the acoustic resonance frequency in the following form for a = b:

$$f_1^{acoustic} = \frac{\omega_1}{2\pi} = \frac{ck_c}{2\pi} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2} = \frac{c}{2a}\sqrt{m^2 + n^2} \qquad (143a)$$

Substituting $\bar{c}$ = 343.8 m/sec and a = 18" = 0.4572 m, one obtains from Eqn. (143a) for the acoustic resonance frequencies:

$$f_{m,n}^{acoustic} = 375.98\sqrt{m^2 + n^2} \cong 376.0\sqrt{m^2 + n^2} \qquad (143b)$$

In the following Table D the theoretical numerical values of Eqn. (143) are arranged in order of increasing acoustic resonance frequencies $f_{m,n}^{acoustic}$. The corresponding resonance frequency experimental data of the upward spikes are given from Figure 1.

TABLE D

ACOUSTIC RESONANCE FREQUENCIES

m - n	$m^2 + n^2$	$f_{m,n}^{acoustic}$	Experimental
*1 - 0	1	376.0	*369
1 - 1	2	531.7	517
**2 - 0	4	752.0	**742
2 - 1	5	840.7	798
*2 - 2	8	1,063.4	*1,080
*3 - 0	9	1,127.9	*1,080
3 - 1	10	1,189.0	1,220
*3 - 2	13	1,355.6	*1,300
4 - 0	16	1,503.9	1,470
4 - 1	17	1,550.2	
3 - 3	18	1,595.1	1,600
*4 - 2	20	1,681.4	*1,720
4 - 3	25	1,879.9	1,830
5 - 0	25	1,879.9	1,830
5 - 1	26	1,917.1	1,940
*5 - 2	29	2,024.7	*2,070
4 - 4	32	2,126.9	
5 - 3	34	2,192.3	2,200
6 - 0	36	2,255.9	
6 - 1	37	2,287.0	

Note: * Denotes major experimental upward spikes

$$f_{m,n}^{acoustic} = 376\sqrt{m^2 + n^2}$$

TABLE D (continued)

m - n	$m^2 + n^2$	$f^{acoustic}_{m,n}$	Experimental
6 - 2	40	2,377.9	
5 - 4	41	2,407.4	
6 - 3	45	2,522.1	
7 - 0	49	2,631.9	2,600
5 - 5	50	2,658.6	2,600
7 - 1	50	2,658.6	2,600
6 - 4	52	2,711.2	
7 - 2	53	2,737.2	
7 - 3	58	2,863.4	2,900
6 - 5	61	2,936.5	2,900
8 - 0	64	3,007.8	
7 - 4	65	3,031.3	
8 - 1	65	3,031.3	
8 - 2	68	3,100.4	
6 - 6	72	3,190.3	3,180
8 - 3	73	3,212.4	3,180
7 - 5	74	3,234.3	
8 - 4	80	3,362.9	
9 - 0	81	3,383.8	
9 - 1	82	3,404.6	
**7 - 6	85	3,466.3	**3,500
**9 - 2	85	3,466.3	**3,500
**8 - 5	89	3,547.0	**3,500
9 - 3	90	3,566.8	
9 - 4	97	3,703.0	

TABLE D (continued)

m - n	$m^2 + n^2$	$f_{m,n}^{acoustic}$	Experimental
7 - 7	98	3,722.0	
8 - 6	100	3,759.8	3,760
10 - 0	100	3,759.8	3,760
10 - 1	101	3,778.6	3,760
10 - 2	104	3,834.2	
9 - 5	106	3,871.1	
10 - 3	109	3,925.2	
*8 - 7	113	3,996.7	*4,000
*10 - 4	116	4,049.3	*4,000
9 - 6	117	4,067.0	4,070
11 - 0	121	4,135.8	4,070
11 - 1	122	4,152.7	
10 - 5	125	4,203.5	
11 - 2	125	4,203.5	
8 - 8	128	4,253.8	
9 - 7	130	4,286.9	
11 - 3	130	4,286.9	
10 - 6	136	4,384.7	4,400
11 - 4	137	4,400.8	4,400
*12 - 0	144	4,511.8	*4,480
*9 - 8	145	4,527.6	*4,480
*12 - 1	145	4,527.6	*4,480
11 - 5	146	4,543.0	
12 - 2	148	4,574.2	
10 - 7	149	4,589.6	

TABLE D (continued)

m - n	$m^2 + n^2$	$f^{acoustic}_{m,n}$	Experimental
12 - 3	153	4,650.5	
11 - 6	157	4,711.0	
12 - 4	160	4,755.8	
9 - 9	162	4,785.5	
10 - 8	164	4,814.8	
12 - 5	169	4,887.7	4,860
13 - 0	169	4,887.7	4,860
11 - 7	170	4,902.0	
13 - 1	170	4,902.0	
13 - 2	173	4,945.3	
13 - 3	178	5,016.3	
12 - 6	180	5,044.1	
10 - 9	181	5,058.4	
11 - 8	185	5,113.7	
13 - 4	185	5,113.7	
12 - 7	193	5,223.1	
13 - 5	194	5,236.6	
14 - 0	196	5,263.7	

$$f^{acoustic}_{m,n} = 376\sqrt{m^2 + m^2}$$

Particular large sharp upward spikes of this phenomena appear at $f^{acoustic}_{m,n}$ for (m,n) = (1,0), (2,0), (2,2), (3,0), (3,2), (4,2), (5,2), (7,6), (9,2), (8,5), (8,7), (10,4), (12,0), (9,8), (12,1) which are denoted by one star at Table D. Particular prominent large sharp upward spikes appear at (2,0) and at (7,6), (9,2), (8,5) which are denoted by two stars at Table D. The first large sharp spike at (2,0) of the acoustic resonance frequency is explained as excited by the basic even particularly large vibrations modes (2,0) and (0,2) of the aluminum panel. The other particularly large sharp upward spike of the acoustic resonance is explained as the interaction of the acoustic modes (7,6), (9,2), (8,5) with the close by cavity resonance mode s = 52.

E. <u>The Plate Resonance</u>

It has been shown in Chapter VIII that the free vibrations of the panel will establish plate acoustic wave resonance modes (m,n) of circular frequency ω_2 and ω_3 in the rigid duct, the value of which is given in Eqn. (123) and Eqn. (124). These acoustic wave plate resonance modes have a frequency which is much below cut-off of the corresponding higher order modes in the duct, and will not propagate at all. Their effect is primarily a near zone effect near the plate. These plate resonance frequencies will also interact with the plane acoustic wave excited by the plate, and this plane acoustic wave could propagate in the duct at all frequencies since its cut-off frequency in the rigid duct is zero. By these two aspects, the plate resonance modes will affect the microphones. The experimental results in Figure 1 show that this plane acoustic wave, which results from the interaction with the plate

resonance modes, is added in the transmitter microphone out of phase to the incident acoustic wave, and is added in the receiver microphone in phase to the incident acoustic wave. As a result, the pressure in the receiver microphone will be higher than usual at plate resonance frequencies, and the pressure at the transmitter microphone will be lower than usual at plate resonance frequencies. This will result in less "noise reduction" in the experimental data at plate resonance frequencies, and thus will result in downward spikes at plate resonance frequencies in the experimental data given in Figure 1.

Equation (121b) gives the plate resonance frequencies in the following form for a = b:

$$f_2 = f_3 = \frac{1}{2\pi}\sqrt{\frac{D}{\rho_p h}}\, k_c^{\,2} = \frac{1}{2\pi}\sqrt{\frac{D}{\rho_p h}}\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{a}\right)^2\right] =$$

$$= \frac{\pi}{2a^2}\sqrt{\frac{D}{\rho_p h}}\,(m^2 + n^2) \tag{144a}$$

Substituting the value of $D = \dfrac{Eh^3}{12(1 - \nu^2)}$ in Eqn. (144a), one obtains:

$$f_2^{plate} = f_3^{plate} = \frac{\pi h}{2a^2}\sqrt{\frac{E}{12\rho_p(1 - \nu^2)}}\,(m^2 + n^2) \tag{144b}$$

Using the numerical values for the present case from the beginning of this chapter, one obtains for the plate resonance frequencies:

$$f_{m,n}^{plate} = 7.353\,(m^2 + n^2) \tag{144c}$$

In the following Table E the theoretical numerical values of Eqn. (144) are arranged in order of increasing plate resonance frequencies

$f^{plate}_{2m,n} = f^{plate}_{3m,n}$. The corresponding resonance frequency experimental data of the downward spikes are given from Figure 1. The resonance frequency where particularly large downward spikes are found in Figure 1 are denoted by a star. The even modes of the plate vibrations are found most prominent in (2,2), (4,4), (6,4), (8,8) of the first few modes. The plate resonance mode (6,6) downward spike is cancelled by the acoustic resonance upward spike of mode (1,1). The plate resonance mode (10,10) downward spike is cancelled by the acoustic resonance upward spike of mode (4,0).

TABLE E

PLATE RESONANCE FREQUENCIES

m - n	$m^2 + n^2$	$f^{plate}_{m,n}$	Experimental
2 - 0	4	29.4	
*2 - 2	8	58.8	*63
4 - 0	16	117.6	102
4 - 2	20	147.1	134
*4 - 4	32	235.3	*219
6 - 0	36	264.7	247
6 - 2	40	294.1	290
*6 - 4	52	382.4	*383
8 - 0	64	470.6	454
8 - 2	68	500.0	

Note: * Denotes major experimental downward spikes

$$f^{plate}_{m,n} = 7.353\ (m^2 + n^2)$$

TABLE E (continued)

m - n	$m^2 + n^2$	$f_{m,n}^{plate}$	Experimental
6 - 6	72	529.4	517
8 - 4	80	588.2	
8 - 6	100	735.3	
10 - 0	100	735.3	
10 - 2	104	764.7	780
10 - 4	116	852.9	
*8 - 8	128	941.2	*918
10 - 6	136	1,000.0	970
12 - 0	144	1,058.8	1,040
12 - 2	148	1,088.2	
12 - 4	160	1,176.5	
10 - 8	164	1,205.9	1,200
12 - 6	180	1,323.5	
14 - 0	196	1,441.2	
10 - 10	200	1,470.6	
14 - 2	200	1,470.6	
12 - 8	208	1,529.4	
*14 - 4	212	1,558.8	*1,540
*14 - 6	232	1,705.9	*1,640
12 - 10	244	1,794.1	1,770
16 - 0	256	1,882.4	1,890
14 - 8	260	1,911.8	1,890
16 - 2	260	1,911.8	
16 - 4	272	2,000.0	2,000
12 - 12	288	2,117.7	

TABLE E (continued)

m - n	$m^2 + n^2$	$f^{plate}_{m,n}$	Experimental
16 - 6	292	2,147.1	
14 - 10	296	2,176.5	
16 - 8	320	2,353.0	
18 - 0	324	2,382.4	
18 - 2	328	2,411.8	
*14 - 12	340	2,500.0	*2,420
*18 - 4	340	2,500.0	*2,420
16 - 10	356	2,617.7	
*18 - 6	360	2,647.1	*2,630 - 2,760
18 - 8	388	2,853.0	
14 - 14	392	2,882.4	
16 - 12	400	2,941.2	
20 - 0	400	2,941.2	
20 - 2	404	2,970.6	
*20 - 4	416	3,058.8	*3,000
18 - 10	424	3,117.7	
20 - 6	436	3,205.9	
*16 - 14	452	3,323.6	*3,280
20 - 8	464	3,411.8	
18 - 12	468	3,441.2	
22 - 0	484	3,558.9	
22 - 2	488	3,588.3	
20 - 10	500	3,676.5	
22 - 4	500	3,676.5	
16 - 16	512	3,764.7	3,700 - 3,800

TABLE E (continued)

m - n	$m^2 + n^2$	$f^{plate}_{m,n}$	Experimental
18 - 14	520	3,823.6	3,700 - 3,800
22 - 6	520	3,823.6	3,700 - 3,800
*20 - 12	544	4,000.0	*3,920
22 - 8	548	4,029.4	4,060
24 - 0	576	4,235.3	
18 - 16	580	4,264.7	
24 - 2	580	4,264.7	
*22 - 10	584	4,294.2	*4,320
*24 - 4	592	4,353.0	*4,320
20 - 14	596	4,382.4	
*24 - 6	612	4,500.0	*4,510 - 4,600
*22 - 12	628	4,617.7	*4,510 - 4,600
24 - 8	640	4,705.9	
18 - 18	648	4,764.7	
20 - 16	656	4,823.6	
24 - 10	676	4,970.6	4,930
26 - 0	676	4,970.6	
22 - 14	680	5,000.0	
26 - 2	680	5,000.0	
26 - 4	692	5,088.3	
26 - 6	712	5,235.3	
24 - 12	720	5,294.2	
20 - 18	724	5,323.6	

It has been shown in Eqns. (123) and (124) that the plate resonance frequencies are split $\omega_2 \neq \omega_3$ by a small value to two components, similar to the split of the spectrum of the atom by applying magnetic fields. As may be seen from Eqn. (124c) the difference between the frequencies $\omega_2 - \omega_3$ is proportional to k_c and becomes larger for high mode numbers (m,n). Thus, we see in Figure 1 that as a result, the downward resonance spikes resulting from the plate resonance frequencies are duller in most cases than the upward spikes, becoming duller for higher mode numbers and higher resonance frequencies. Because of the logarithmic scale, narrowing the distinction at higher frequencies, some of the higher frequencies downward spikes will look much duller at higher frequencies under a regular scale of frequencies.

It should be pointed out that because of the interaction of the above three major effects, the cavity resonance, the acoustic resonance and the plate resonance, sometimes the different resonance effects add out of phase and cancel each other, leaving no spikes in Figure 1, but only a break in the resonance curve, if at all. Thus, it is sometimes impossible to detect a particular frequency in the experimental results of Figure 1, because it has been cancelled by other resonance phenomena. Sometimes these different resonance phenomena interact with each other to produce a resonance in the experimental curve in Figure 1 at frequencies which can not be identified with any one of the single phenomena discussed above.

F. The Wooden Back Panel Resonance

Although the wooden back panel is relatively massive, it can act as a resonator under the influence of the plane acoustic wave incident upon it. Thus, the wooden back panel excites reflected plane acoustic waves at frequencies of its own wooden "plate" resonance, similarly to the phenomena discussed in the previous section. Rewriting Eqn. (144b):

$$f_{m,n}^{wood} = \frac{\pi h}{2a^2} \sqrt{\frac{E}{12\rho_p(1 - \nu^2)}} \, (m^2 + n^2) \tag{145a}$$

and substituting the numerical values for the wooden back panel given in Eqn. (135), one obtains:

$$f_{m,n}^{wood} = 155 \; (m^2 + n^2) \tag{145b}$$

In the following Table F the theoretical numerical values of Eqn. (145) are arranged in order of increasing wooden back panel "plate" resonance frequencies $f_{m,n}^{wood}$. The corresponding resonance frequency experimental data of the downward spikes are given from Figure 1. Particularly large downward spikes are denoted by a star.

TABLE F

WOOD RESONANCE FREQUENCIES

m - n	$m^2 + n^2$	$f_{m,n}$	Experimental $f_{m,n}$
*2 - 0	4	620	*630
**2 - 2	8	1,240	**1,260
**4 - 0	16	2,480	**2,420
*4 - 2	20	3,100	*3,010
4 - 4	32	4,960	4,930
6 - 0	36	5,580	

Note: * Denotes major experimental downward spikes

$$f_{m,n} = 155\ (m^2 + n^2)$$

Of particular interest are the large downward spikes of $f_{2,2}^{wood}$ and $f_{4,0}^{wood}$. The resonance frequency $f_{2,2}^{wood}$ = 1,240 Hz does not have any plate resonance frequency close by to interact strongly with it, and therefore it is a sharp downward spike. The resonance frequency $f_{4,0}^{wood}$ = 2,480 Hz interacts strongly with the aluminum plate resonance frequencies $f_{14,12}^{plate} = f_{18,4}^{plate}$ = 2,500 Hz and therefore the downward spike in this case is a dull one.

CHAPTER XI

SUMMARY

In the present report the theoretical background has been given and the theory has been developed for acoustic plane waves normally incident on a clamped panel in a rectangular duct. The coupling theory between the elastic vibrations of the panel (plate) and the acoustic waves propagation in infinite space and in the duct has been considered in detail. The coupling theory developed in this report is based on the theory of acoustic waves propagation and the dynamic theory of elasticity and the plate vibrations. The partial differential equation which governs the vibrations of the panel (plate) has been modified by adding to it stiffness (spring) forces and damping forces, and the fundamental resonance frequency f_o and the attenuation factor α are discussed in detail.

The noise reduction expression based on the present theory has been found to agree well with the corresponding experimental data of a sample aluminum panel in the mass controlled region ($f > f_o$), the damping controlled region ($f \sim f_o$) and the stiffness controlled region ($f < f_o$). All the frequency positions of the upward and downward resonance spikes in the sample experimental data are identified theoretically as resulting from four cross-interacting major resonance phenomena: The cavity resonance, the acoustic resonance, the plate resonance, the wooden back panel resonance, and detailed tables are given for the values of these resonance frequencies for each case.

In Chapter I the basic general theoretical considerations are introduced.

In Chapter II some basic concepts of the theory of propagation of acoustic waves and the partial differential equations which govern the propagation of acoustic waves in three dimensions are given.

In Chapter III the acoustic uniform plane wave which varies harmonically with time is given using complex phasor representation. The incident, reflected and standing acoustic waves as well as the reflection coefficient are discussed.

In Chapter IV the porpagation of acoustic waves in rectangular ducts is discussed in detail and some of the basic concepts are introduced. The fundamental acoustic mode, which behaves like a plane acoustic wave in infinite space, propagates in the duct at all frequencies. Higher order acoustic modes attenuate exponentially at frequencies below the corresponding cut off frequencies, and propagate at frequencies above the cut off frequencies. A list of the cut off frequencies for a duct with a square cross-section is given.

In Chapter V some basic concepts of the vibration theory of elastic plates, as based on the theory of elasticity, and the partial differential equation which governs such vibrations, are discussed. The different possible boundary conditions, and the characteristic frequencies of vibration are given.

In Chapter VI the boundary value problem of acoustic plane wave normally incident on an infinite vibrating plate is solved, and the transmission and reflection coefficients of the corresponding acoustic waves are found. The transmission loss coefficient, the noise reduction coefficient, and the insertion loss coefficient are defined and are related to each other. The solution gives the "normal incidence mass law."

This theoretical result agrees with the experimental results in the mass controlled region of high frequencies, but does not explain the experimental results in the damping contolled region (near the fundamental resonance frequency) or in the stiffness controlled region (frequencies below the fundamental resonance frequency).

In Chapter VII the boundary value problem of an acoustic plane wave normally incident on a finite clamped panel is solved. The partial differential equation which governs the vibrations of the plate (panel) is modified by adding to it stiffness (spring) forces and damping forces, and the fundamental resonance frequency is defined. The transmission and reflection coefficients of the corresponding acoustic waves are found. The transmission loss coefficient, the noise reduction coefficient and insertion loss coefficients are derived. This theoretical result agrees with the experimental results in the mass controlled region (frequencies above the fundamental resonance frequency) as well as in the damping controlled region (near the fundamental resonance frequency) and in the stiffness controlled region (frequencies below the fundamental resonance frequency). A formula is given for the calculation of the damping factor from the experimental curve value at the fundamental resonance frequency.

In Chapter VIII the resonance frequencies excited by the free vibrations of the finite plate (panel) and its coupling with the air surrounding it is discussed. A cubic equation for $(2\pi f)^2$, where f are the resonance frequencies for each mode (m,n) of vibration, is solved. One solution is found to be the acoustic resonance frequency f_1, which is slightly smaller than the cut-off frequency of the corresponding

mode in the rigid duct, and therefore it will not propagate, but will attenuate exponentially. The other two solutions are found to be the plate resonance frequencies $f_2 \sim f_3$, which differ in magnitude slightly, and are very much smaller than the cut-off frequency of the corresponding mode in the rigid duct. These two close together plate resonance frequencies will excite higher order acoustic modes which will effect the experimental results only in the near zone, and will also interact with the plane acoustic wave fundamental mode, which can propagate in the rigid duct at any frequency.

In Chapter IX the propagation of the plane acoustic wave in the receiving chamber of the Beranek tube is discussed. It is shown that the reflection of the acoustic plane wave from the wooden back panel will cause standing waves and the cavity resonance phenomena. The vibrations of the wooden back panel as a vibrating plate is discussed. The effect of the coupling between two resonance systems on the resonance frequency of each system by itself is considered.

In Chapter X the experimental results are discussed in detail in view of the theory presented in this report. Six major theoretical apsects of the experimental results are presented:

A. Plane Wave on the Finite Panel

The theoretical noise reduction curve is calculated and drawn in heavy line in Figure 1 and follows closely the average experimental data in the mass controlled region, the damping controlled region and the stiffness controlled region. The difference between the wiggly experimental curve and the heavy line theoretical curve represents the effects of the resonance phenomena and the higher order modes.

B. <u>The Fundamental Resonance Frequency (f_o = 63 Hz)</u>

The fundamental resonance frequency f_o = 63 Hz is the result of the interaction between the basic cavity resonance frequency f_1^{cavity} = 68 Hz of the standing plane wave along the length of the Beranek tube, and the first even resonance mode of the aluminum panel vibrations, which excite the acoustic waves at $f_{2,2}^{plate}$ = 58.8 Hz.

C. <u>The Cavity Resonance</u>

A large number of the experimental upward spikes in the experimental curve in Figure 1 are identified with the theoretical resonance frequencies of the standing plane acoustic wave reflected from the wooden back panel. A complete list of these cavity resonance frequencies is given.

D. <u>The Acoustic Resonance</u>

The rest of the experimental upward spikes in the experimental curve in Figure 1 are identified with the theoretical resonance frequencies of the acoustic resonance modes of the panel vibrations. A complete list of these acoustic resonance frequencies is given.

E. <u>The Plate Resonance</u>

The experimental downward spikes (with the exception of one) in the experimental curve in Figure 1 are identified with the theoretical resonance frequencies of the plate resonance modes of the panel vibrations. A complete list of these acoustic resonance frequencies is given.

F. <u>The Wooden Back Panel Resonance</u>

The two most prominent downward spikes in the experimental curve in Figure 1 are identified with the theoretical resonance frequencies of the "plate" resonance frequencies of the wooden back panel. The first sharp downward spike has not been identified before.

The detailed features of the experimental curve in Figure 1 have been explained by using the six major theoretical aspects of the experimental results discussed above, and the theoretical results agree with experiment.

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